

## The Department of Mathematics

2020–21–B term

**Course Name** Algebraic Geometry - Schemes 2

**Course Number** 201.2.2121

**Course web page**

[https://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-2/course\\_page.html](https://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-2/course_page.html)

**Lecturer** Prof. Amnon Yekutieli, <amyekut@bgu.ac.il>, Office 202

**Office Hours** <https://www.math.bgu.ac.il/en/teaching/hours>

### Abstract

See pdf files

### Requirements and grading<sup>1</sup>

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<sup>1</sup>Information may change during the first two weeks of the term. Please consult the webpage for updates



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המחלקה למתמטיקה  
אוניברסיטת בן גוריון  
באר שבע 84105

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## Algebraic Geometry – Schemes 2

BGU, Spring 2020-21

Catalogue no. 201.2.2121

**General Information.** The course will be in English. It is a continuation of "[Algebraic Geometry – Schemes 1](#)" from the fall semester.

The course web page (with up-to-date information):

[https://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-2/course\\_page.html](https://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-2/course_page.html)

Participants from outside the BGU community (and from all over the world) are welcome. Please distribute this page to interested participants.

See the "[first day handout](#)" on the course page for technical stuff.

**Prerequisites and Level.** The course is intended for graduate students and advanced undergraduate students.

Participants of the course should have good knowledge of these topics: the material of the course "[Schemes 1](#)" (i.e. categories and functors, sheaves, ringed spaces and their maps and sheaf operations); some classical algebraic geometry (varieties over an algebraically closed field, or at least algebraic curves); commutative algebra; and some Galois theory. It will be very useful to also know some homological algebra, algebraic topology, and the geometry of real differentiable or complex analytic manifolds.

The level of the course will be calibrated – in terms of rate of progress and sophistication of the presentation – to the audience. I hope that we can go faster than in the first semester (since gaps in preparation have been filled).

Please send me an email if you are considering attending the course, indicating which of the topics above you have learned (in a formal course or privately), your academic status (degree, year and institution), and whether you intend to register or just to listen.

**Course Topics:** (tentative)

1. **Locally Ringed Spaces.** Recalling ringed spaces, sheaves of modules, maps of ringed spaces and sheaf operations. Locally ringed spaces and their maps. Examples from differential and classical algebraic geometry. Locally free sheaves, vector bundles, Picard group.
2. **Affine Schemes.** Definitions and basic properties. Examples from classical algebraic geometry and

- from arithmetic. Noetherian affine schemes. Coherent and quasi-coherent sheaves. Maps of affine schemes, closed subschemes, principal open subschemes.
3. **Schemes and their Maps.** Definitions and basic properties. Noetherian schemes. Examples from classical algebraic geometry. Coherent and quasi-coherent sheaves. Fiber products and base change. Finite, finite type, flat, separated and proper maps of schemes.
  4. **Projective Spaces.** Definition of the projective space  $\mathbf{P}_{\mathbb{K}}^n$ . Line bundles and maps to projective space. Proving that projective maps are proper. Computing the Picard group of  $\mathbf{P}_{\mathbb{K}}^n$ . Blowups and some birational geometry.
  5. **Sheaf Cohomology.** Nonabelian 1-st Čech cohomology, abelian sheaf cohomology (Čech and derived functor), Serre Duality, Riemann-Roch for curves, etc.
  6. **Algebraic Differential Calculus.** Smooth and étale maps, differential forms, differential operators, etc.
  7. **The Functor of Points and Moduli Spaces.** Yoneda embedding. Examples of moduli spaces. A first glance at algebraic spaces and algebraic stacks.
  8. **Group Schemes and their Lie Algebras.** Definitions and basic properties. Examples of affine group schemes (tori,  $\mathrm{GL}_{n,\mathbb{K}}$ ), abelian varieties. The Lie algebra functor.

### Bibliography:

1. [Course lecture notes](#) (to be posted weekly).
2. Hartshorne, "Algebraic Geometry", Springer.
3. Matsumura, "Commutative Ring Theory", Cambridge U. Press, 1989.
4. Altman and Kleiman, "A Term of Commutative Algebra" ([free online book](#))
5. Kashiwara and Schapira, "Sheaves on Manifolds", Springer.
6. de Jong (Ed.), "The Stacks Project", <https://stacks.math.columbia.edu>.
7. Olsson, "Algebraic Spaces and Stacks", AMS.
8. Eisenbud and Harris, "The Geometry of Schemes", Springer.



**גיאומטריה אלגברית – סכמות 2**  
**Algebraic Geometry – Schemes 2**  
**201-2-2121**

**סמסטר ב' (אביב) תשפ"א    Semester B (Spring) 2020-21**

<i>lecturer</i>	Prof. Amnon Yekutieli	פרופ' אמנון יקותיאלי	מרצה
<i>email</i>	<a href="mailto:amyekut@math.bgu.ac.il">amyekut@math.bgu.ac.il</a>		אימייל
<i>web page</i>	<a href="http://www.math.bgu.ac.il/~amyekut">http://www.math.bgu.ac.il/~amyekut</a>		דף ברשת
<i>office</i>	email		משרד
<i>office hours</i>	email		שעות קבלה
<i>time of lectures</i>	Wednesday 12:00-14:00	רביעי 12:00-14:00	שעות ההרצאות
<i>place of lectures</i>	<b>Zoom</b> software	תוכנת <b>זום</b>	מקום ההרצאות
<i>course web page</i>	<a href="http://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-2/course_page.html">~amyekut/teaching/2020-21/schemes-2/course_page.html</a>		אתר ברשת
<i>material and bibliography</i>	See syllabus	ראה סילבוס	חומר הלימוד וספרות
<i>teaching language</i>	English	אנגלית	שפת הלימוד
<i>course grade</i>	Pass/fail. Passing the course requires attending all lectures (on zoom) and submitting most of the HW.	מילולי (עובר/נכשל). קבלת ציון עובר מחייבת נוכחות בכל ההרצאות (דרך זום) והגשת רוב שעורי הבית.	ציון הקורס
<i>homework</i>	Assigned every week. Checking will be done sporadically.	יוטלו כל שבוע, ויבדקו מדי פעם באופן מדגמי.	שעורי בית



## Course topics

1. Sheaves on topological spaces
2. Affine schemes
3. Schemes and morphisms between them.
4. Quasi-coherent sheaves
5. Separated and proper morphisms.
6. Vector bundles and the Picard group of a scheme.
7. The functor of points and moduli spaces.
8. Morphisms to projective space and blow-ups.
9. Smooth morphisms and differential forms.
10. Sheaf cohomology.
11. Group schemes.