Abstract

Ordered sets and well ordered sets. Ordinals. Linearly ordered sets. Uniqueness of countable linear orders without endpoints.

The set of finite ordinals, construction of the natural numbers, the induction principle and some of its equivalents.

Countable sets, construction of the rational numbers.

Construction of the real field.

Cardinality, cardinals, and the Cantor-Bernstein theorem.

Uncountable sets, Cantor’s theorem, applications.

The axiom of choice and its equivalents (the well ordering principle, Zorn’s lemma).

Applications of the axiom of choice. Transfinite induction.

Throughout the course we will see applications of the course’ material in algebra, logic, graph theory, Euclidean spaces and infinite combinatorics.

Requirements and grading

90% final exam and 10% HW assignments. A passing grade in 4/6 assignments is a prerequisite for obtaining a grade in the course.

\footnote{Information may change during the first two weeks of the term. Please consult the webpage for updates}
Course topics


2. Comparison of sets. The definition of cardinality as an equivalence class over equinumerosity. The Cantor-Bernstein theorem. Cantor’s theorem on the cardinality of the power-set.


5. The construction of the ordered real line as a quotient over Cauchy sequences of rationals.

6. Konig’s lemma on countably infinite trees with finite levels. Applications. A countable graph is k-colorable iff every finite subgraph of it is k-colorable.

7. Well ordering. Isomorphisms between well-ordered sets. The axiom of choice formulated as the well-ordering principle. Example. Applications. An arbitrary graph is k-colorable iff every finite subgraph is k-colorable.


11. Infinite cardinals as initial ordinals. Basic cardinal arithmetic. Cardinalities of well known sets. Continuous real functions, all real functions, the automorphisms of the real field (with and without order).