

The Department of Mathematics

2021–22–B term

Course Name Introduction to representation theory of groups

Course Number 201.1.0511

Course web page

<https://www.math.bgu.ac.il/en/teaching/spring2022/courses/representation-theory>

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Office Hours <https://www.math.bgu.ac.il/en/teaching/hours>

Abstract

Requirements and grading¹

Course topics

1. Introduction: Actions of groups on sets. Induced linear actions. Multilinear algebra.
2. Representations of groups, direct sum. Irreducible representations, semi-simple representations. Schur's lemma. Irreducible representations of finite abelian groups. Complete reducibility, Maschke's theorem.
3. Equivalent representations. Morphisms between representations. The category of representations of a finite group. A description using the group ring. Multilinear algebra of representations: dual representation, tensor product (inner and outer).
4. Decomposition of the regular representation into irreducible representations. The number of irreducibles is equal to the number of conjugacy classes. Matrix coefficients, characters, orthogonality.
5. Harmonic analysis: Fourier transform on finite groups and the non-commutative Fourier transform.

¹Information may change during the first two weeks of the term. Please consult the webpage for updates



6. Frobenius divisibility and Burnside $p^a q^b$ theorem.
7. Constructions of representations: induced representations. Frobenius reciprocity. The character of induced representation. Mackey's formula. Mackey's method for representations of semi-direct products.
8. Induction functor: as adjoint to restrictions, relation to tensor product. Restriction problems, multiplicity problems, Gelfand pairs and relative representation theory.
9. Examples of representations of specific groups: SL_2 over finite fields, Icosahedron group, Symmetric groups.
10. Artin and Brauer Theorems on monomial representations