אוניברסיטת
בן-גוריון בנגב

The Department of Mathematics

2022—23—B term

Course Name Introduction to Algebraic Geometry

Course Number 201.2.4111

Course web page
`https://sites.google.com/view/amyekut-math-bgu/home/teaching/intro-ag_2022-23`

Lecturer Prof. Amnon Yekutieli, <amyekut@bgu.ac.il>, Office 202

Office Hours `https://www.math.bgu.ac.il/en/teaching/hours`

Abstract

See course **syllabus** and **administrative information** on the course web page.[1] These pdf files are also uploaded here.

My active web page is this (on Google Drive)[2], not the one you see when you click my name above.

Please ignore the section "Course Topics" below, fi it appears. (I don't know how to make this text go away – but maybe it did anyhow.)

Requirements and grading[3]

See course **syllabus** and **administrative information** on the course web page.[4] These pdf files are also uploaded here.

My active web page is this (on Google Drive)[5], not the one you see when you click my name above.

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[1] `https://sites.google.com/view/amyekut-math-bgu/home/teaching/intro-ag_2022-23`

[2] `https://sites.google.com/view/amyekut-math-bgu/home`

[3] Information may change during the first two weeks of the term. Please consult the webpage for updates

[4] `https://sites.google.com/view/amyekut-math-bgu/home/teaching/intro-ag_2022-23`

[5] `https://sites.google.com/view/amyekut-math-bgu/home`

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פרופ' אמנון יקותיאלי
המחלקה למתמטיקה
אוניברסיטת בן גוריון
באר שבע 84105

11 January 2023

Graduate Course

Introduction to Algebraic Geometry

Semester B (Spring) 2022-23

Course name: Introduction to Algebraic Geometry

Catalog no: 201.2.4111

Schedule: Semester B (Spring) 2022-23

General Description. Algebraic geometry (AG) is the study of geometric objects, called *algebraic varieties*, defined by *polynomial equations* (e.g. circles and ellipses). Since ancient times mathematicians were interested in the size and shape of the solution sets of polynomial equations, classifying them, learning how they intersect, etc. During the 19th century AG became intertwined with *complex analytic geometry* and with *number theory*, and later also with *algebraic topology*. A giant leap in AG occurred early in the 20th century, with the emergence of *commutative algebra* as the basic tool for treating algebraic varieties. Around 1960 there was another revolution in AG: the introduction (by Grothendieck) of *schemes*, which are algebro-geometric objects that are much more general than varieties, and can very effectively describe and analyze questions arising from number theory, and also permit an algebro-geometric version of differential calculus. In the 21st century we see further progress in the foundations of AG: it is merging with modern algebraic topology, and goes under the name *derived algebraic geometry*.

Besides being a very interesting and vast mathematical theory on its own, AG is essential, as a working tool, for several other mathematical areas. Among them are *representations of groups and Lie algebras* (currently the Langlands program is the most active research in this direction), *number theory* (notably the proof by Wiles in the 1990s of the Fermat Conjecture, which relied mainly on the study of algebro-geometric objects called elliptic curves), *homotopy theory* (e.g. chromatic homotopy, that also involves elliptic curves), and *mathematical physics* (mirror symmetry, string theory, quantum field theories, mathematical Feynman integrals, much due to Kontsevich). In conjunction with the development of AG, there was rapid development of *homological algebra*, including the theory of *derived categories* (by Grothendieck and his students). Lastly, from a general pedagogical viewpoint, a working knowledge of AG is useful for gaining a deeper understanding of a multitude of other mathematical areas.

This is a course on algebraic varieties over an algebraically closed field. Technically it is much easier than a course on schemes, but it does not touch upon arithmetic geometry. We shall cover most of the standard material, with some additional glances into more advanced or specialized topics. The topics listed below will be adapted – to some extent – to the background and capabilities of the registered students.

spaces, then as differentiable manifolds, and then as ringed spaces. A bit later on we will return to the algebraic varieties $\mathbf{P}^n(\mathbb{K})$, where now $\mathbb{K}$ is an algebraically closed field.

Audience: The course is intended for graduate students at BGU. Strong undergraduate students, and outside students, can attend with special permission by the lecturer.

Organization: The course will consist of 2 weekly lectures (2 hours each), and homework.

Time: **Monday 14:00 - 16:00 and Wednesday 14:00 - 16:00.**

Place: **Broadcast on Zoom**, with projection and student gathering in room 58-201.

Grades: The course grades are pass/fail. Passing requires attending all lectures and submitting almost all the homework assignments. Homework will be assigned each week.

Course web page:
`https://sites.google.com/view/amyekut-math-bgu/home/teaching/intro-ag_2022-23`

Language: English.

Prerequisite courses:
1. "Introduction to Commutative Algebra" or "Commutative Algebra"
2. "Introduction to Topology"

Recommended knowledge: Differentiable manifolds; Galois theory; algebraic topology; homological algebra; complex analysis.

Course Topics: (depending on rate of progress)

1. **Projective space over the real numbers – a warm-up.** We will learn about the projective spaces $\mathbf{P}^n(\mathbb{R})$, first as topological spaces, then as differentiable manifolds, and then as ringed spaces.
2. **Ringed spaces.** Topological spaces equipped with sheaves of rings of functions. Maps between ringed spaces. Study familiar examples: topological spaces and differentiable manifolds.
3. **Categories and functors.** A systematic but gradual study, mixed in with the other topics.
4. **Recalling commutative algebra.** Noetherian rings, Hilbert Basis Theorem, algebraic and transcendental field extensions, Hilbert Nullstellensatz. Localization of rings, prime ideals, flatness. Tensor products of rings.
5. **Affine algebraic varieties.** (All varieties are over an algebraically closed field.) The Zariski topology and the sheaf of algebraic functions. Maps of affine varieties, fibered products, subvarieties, local rings. Dimension of an affine variety and the transcendence degree of its function field.
6. **Projective algebraic varieties.** Definition by quotients, by homogeneous coordinate rings and by gluing. Fibered products of projective varieties. Quasi-projective varieties.
7. **Algebraic varieties.** Gluing, the separation condition, algebraic groups (brief).
8. **Types of maps between varieties**. Finite, quasi-finite, flat, affine and projective maps.
9. **The classification of curves.** Nonsingular projective curves and their functions fields. Elliptic curves (brief).
10. **Enumerative geometry**. Bézout Theorem. Some intersection theory. A brief survey of modern methods.
11. **Vector bundles**. Definition, examples. Gluing vector bundles.
12. **Line bundles and the Picard group**. Ample line bundles and morphisms to projective space. The

13. **Birational geometry**. (Brief) Blowups. Some examples. Singularities and their resolutions. A brief survey of modern methods.
14. **Sheaves of modules.** (Brief) From locally free sheaves to vector bundles and back. Coherent sheaves. Serre's Theorem on affine open sets.
15. **Sheaf cohomology.** (Brief) Riemann-Roch Theorem for curves. A quick tour of Serre Duality.
16. **Algebraic groups.** (Brief) Some examples. Lie algebras. Group actions, torsors, quotients.
17. **Differential algebraic geometry.** (Brief) The tangent bundle, derivations, differential forms. Étale and smooth maps.

Course Notes: Typed course notes will be available every week on the course web page. These will be based partly on the books listed below, and mostly on the lecturer's preference of teaching. The homework exercises will be included in the notes.

Bibliography.

1. Hartshorne, "Algebraic Geometry", Springer, 1977.
2. Gathmann, "Algebraic Geometry" (2022), free online book.
3. Reid, "Undergraduate Algebraic Geometry" (2013), free online book.
4. Liu, "Algebraic Geometry and Arithmetic Curves", Oxford, 2002.
5. Vakil, "THE RISING SEA – Foundations of Algebraic Geometry" (2017), free online book.
6. Altman and Kleiman, "A Term of Commutative Algebra" (2021), free online book.
7. D. Eisenbud, "Commutative Algebra", GTM 150, Springer, 1995.
8. Course notes, updated weekly, on the course web page.

First Day Handout | דף מידע לתלמיד 11/01/2023

מבוא לגיאומטריה אלגברית
Introductiion to Algebraic Geometry
201.2.4111

Semester B (Spring) 2022-23 סמסטר ב' (אביב) תשפ"ג

lecturer	Prof. Amnon Yekutieli	פרופ' אמנון יקותיאלי	*מרצה*
email	amyekut@math.bgu.ac.il		*אימייל*
web page	https://sites.google.com/view/amyekut-math-bgu/home		*דף ברשת*
office	Email (Zoom possible)	באימייל (אפשר שיחת זום)	*משרד*
office hours	By appointment	לפי תיאום	*שעות קבלה*
time of lectures	Monday 14:00-16:00 Wednesday 14:00-16:00	שני 14:00-16:00 רביעי 14:00-16:00	*שעות ההרצאות*
First lecture	8 March 2023	8 במרץ 2023	*הרצאה ראשונה*
place of lectures	**Broadcast live on Zoom** (Zoom link in email from lecturer). Screening for students in room 58-201.	**שידור חי בזום** (קישור: באימייל מהמרצה). הקרנה על מסך לסטודנטים בחדר 58-201.	*מקום ההרצאות*
course web page	https://sites.google.com/view/amyekut-math-bgu/home/teaching/alg-geom-1_2022-23		*האתר של הקורס ברשת*
material and bibliography	See syllabus	ראה סילבוס	*חומר הלימוד וספרות*
teaching language	English	אנגלית	*שפת הלימוד*
course grade	Pass/fail. Passing the course requires attending all lectures (possibly on Zoom) and submitting most of the HW.	מילולי (עובר/נכשל). קבלת ציון עובר מחייבת נוכחות בכל ההרצאות (אפשר דרך זום) והגשת רוב שעורי הבית.	*ציון הקורס*
homework	Assigned every week. Checking will be done sporadically.	יוטלו כל שבוע, ויבדקו מדי פעם באופן מדגמי.	*שעורי בית*

אוניברסיטת
בן-גוריון בנגב

Course topics

Generally speaking, algebraic geometry deals with geometric objects defined by polynomial equations. We will cover topics 1-3 below, and maybe one other topic, depending on the level of the students.

Topics

.1 Topics from commutative algebra.

.2 Algebraic varieties over an algebraically closed field: definitions and main properties.

.3 Curves and their function fields.

.4 Intersection Theory in the projective plane.

.5 Vector bundles and coherent sheaves.

.6 Picard group and projective embeddings.

.7 Schemes: definitions, examples and basic properties.