

The Department of Mathematics

2022–23–B term

Course Name Noncommutative algebra

Course Number 201.2.5121

Course web page

<https://www.math.bgu.ac.il/en/teaching/spring2023/courses/noncommutative-algebra>

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Office Hours <https://www.math.bgu.ac.il/en/teaching/hours>

Abstract

Requirements and grading¹

Course topics

1. Basic Algebraic Structures: rings, modules, algebras, the center, idempotents, group rings
2. Division Rings: the Hamiltonian quaternions, generalized quaternion algebras, division algebras over \mathbb{F}_q , \mathbb{C} , \mathbb{R} , \mathbb{Q} (theorems of Frobenius and Wedderburn), cyclic algebras, the Brauer–Cartan–Hua theorem
3. Simplicity and semi-simplicity: simplicity of algebraic structures, semi-simple modules, semi-simple rings, Maschke’s theorem
4. The Wedderburn–Artin Theory: homomorphisms and direct sums, Schur’s lemma, the Wedderburn–Artin structure theorem, Artinian rings
5. Introduction to Group Representations: representations and characters, applications of the Wedderburn–Artin theory, orthogonality relations, dimensions of irreducible representations, Burnside’s theorem

¹Information may change during the first two weeks of the term. Please consult the webpage for updates



6. Tensor Products: tensor products of modules and algebras, scalar extensions, the Schur index, simplicity and center of tensor products, the Brauer group, the Skolem–Noether theorem, the double centralizer theorem, maximal fields in algebras, reduced norm and trace, crossed products