Department of Mathematics, BGU

Operator Algebras and Operator Theory

On *Monday, November ,27 2017*

At *16:00 − 17:00*

In *101-*

Gregory Marx (BGU)

will talk about

Completely Positive Noncommutative Kernels

Abstract: It is well known that a function $K : \Omega \times \Omega \to \mathcal{L}(\mathcal{Y})$ (where $\mathcal{L}(\mathcal{Y}($ is the set of all bounded linear operators on a Hilbert space$\mathcal{Y}($ being (1) a positive kernel in the sense of Aronszajn (i.e. $\sum_{i,j=1}^{N} \langle K(\omega_i, \omega_j) y_j, y_i \rangle \geq 0$ for all $\omega_1, \ldots, \omega_N \in \Omega$, $y_1, \ldots, y_N \in \mathcal{Y}$, and $N = 1, 2, \ldots$ (is equivalent to (2) K being the reproducing kernel for a reproducing kernel Hilbert space $\mathcal{H}(K)$, and (3) K having a Kolmogorov decomposition $K(\omega, \zeta) = H(\omega)H(\zeta)^*$ for an operator-valued function $H : \Omega \to \mathcal{L}(\mathcal{X}, \mathcal{Y})$ where $\mathcal{X}$ is an auxiliary Hilbert space.

In work with Joe Ball and Victor Vinnikov, we extend this result to the setting of free noncommutative functions with the target set $\mathcal{L}(\mathcal{Y})$ of K replaced by $\mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{Y}))$ where $\mathcal{A}$ is a C^*-algebra. In my talk, I will give a sketch of our proof and discuss some well-known results (e.g. Stinespring's dilation theorem for completely positive maps) which follow as corollaries. With any remaining time, I will talk about applications and more recent related results.