

Department of Mathematics, BGU

AGNT

On Wednesday, January 22, 2020

At 15:00 – 16:15

In -101

AMNON YEKUTIELI (BGU)

will talk about

Commutative DG Rings and their Derived Categories

ABSTRACT: The commutative DG rings in the title are more commonly known as “nonpositive strongly commutative unital differential graded cochain K -algebras”, where K is a commutative base ring. In the literature the standard assumption is that K is a field of characteristic zero - but one of our themes in this talk is that this assumption is superfluous ($K = \mathbb{Z}$ works just as well).

There are two kinds of derived categories related to commutative DG rings. First, given a DG ring A , we can consider $D(A)$, the derived category of DG A -modules, which is a K -linear triangulated category. This story is well understood by now, and I will only mention it briefly.

In this talk we shall consider another kind of derived category. Let $DGRng$ denote the category whose objects are the commutative DG rings (the base K is implicit), and whose morphisms are the DG ring homomorphisms. The derived category of commutative DG rings is the category $D(DGRng)$ gotten by inverting all the quasi-isomorphisms in $DGRng$. (In homotopy theory the convention is to call it the “homotopy category”, but this is an unfortunate historical accident.)

I will define semi-free DG rings, and prove their existence and lifting properties. Then I will introduce the quasi-homotopy relation on DGRng , giving rise to the quotient category $K(\text{DGRng})$, the “genuine” homotopy category. One of the main results is that the canonical functor from $K(\text{DGRng})$ to $D(\text{DGRng})$ is a faithful right Ore localization.

I will conclude with a theorem on the existence of the left derived tensor product inside $D(\text{DGRng})$, and with the pseudofunctor from $D(\text{DGRng})$ to the TrCat , sending a DG ring A to the triangulated category $D(A)$.

Next semester I will talk about the geometrization of these ideas: “The Derived Category of Sheaves of Commutative DG Rings”.