

Department of Mathematics, BGU

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*On Wednesday, January ,22 2020*

*At 15:00 – 16:15*

*In 101-*

Amnon Yekutieli (BGU)

will talk about

## **Commutative DG Rings and their Derived Categories**

Abstract: The commutative DG rings in the title are more commonly known as “nonpositive strongly commutative unital differential graded cochain  $K$ -algebras”, where  $K$  is a commutative base ring. In the literature the standard assumption is that  $K$  is a field of characteristic zero - but one of our themes in this talk is that this assumption is superfluous ( $K = \mathbb{Z}$  works just as well).

There are two kinds of derived categories related to commutative DG rings. First, given a DG ring  $A$ , we can consider  $D(A)$ , the derived category of DG  $A$ -modules, which is a  $K$ -linear triangulated category. This story is well understood by now, and I will only mention it briefly.

In this talk we shall consider another kind of derived category. Let  $\text{DGRng}$  denote the category whose objects are the commutative DG rings (the base  $K$  is implicit), and whose morphisms are the DG ring homomorphisms. The derived category of commutative DG rings is the category  $D(\text{DGRng})$  gotten by inverting

all the quasi-isomorphisms in  $\text{DGRng}$ . (In homotopy theory the convention is to call it the “homotopy category”, but this is an unfortunate historical accident.)

I will define semi-free DG rings, and prove their existence and lifting properties. Then I will introduce the quasi-homotopy relation on  $\text{DGRng}$ , giving rise to the quotient category  $K(\text{DGRng})$ , the “genuine” homotopy category. One of the main results is that the canonical functor from  $K(\text{DGRng})$  to  $D(\text{DGRng})$  is a faithful right Ore localization.

I will conclude with a theorem on the existence of the left derived tensor product inside  $D(\text{DGRng})$ , and with the pseudofunctor from  $D(\text{DGRng})$  to the  $\text{TrCat}$ , sending a DG ring  $A$  to the triangulated category  $D(A)$ .

Next semester I will talk about the geometrization of these ideas: “The Derived Category of Sheaves of Commutative DG Rings”.