

Department of Mathematics, BGU

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# AGNT

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On Tuesday, November 8, 2022

At 12:40 – 13:40

In 201

Yotam Hendel (Université de Lille)

will talk about

## **On uniform number theoretic estimates for fibers of polynomial maps over finite rings of the form $\mathbb{Z}/p^k\mathbb{Z}$**

Abstract: Let  $f: X \rightarrow Y$  be a morphism between smooth, geometrically irreducible  $\mathbb{Z}$ -schemes of finite type. We study the number of solutions  $\#\{x: f(x) = y \pmod{p^k}\}$  for prime  $p$ , positive number  $k$ , and  $y \in Y(\mathbb{Z}/p^k\mathbb{Z})$ , and show that the geometry and singularities of the fibers of  $f$  determine the asymptotic behavior of this quantity as  $p$ ,  $k$  and  $y$  vary.

In particular, we show that  $f: X \rightarrow Y$  is flat with fibers of rational singularities, a property abbreviated (FRS), if and only if  $\#\{x: f(x) = y \pmod{p^k}\}/p^{k(\dim X - \dim Y)}$  is uniformly bounded in  $p$ ,  $k$  and  $y$ . We then consider a natural family of singularity properties, which are variants of the (FRS) property, and provide for each member of this family a number theoretic characterization using the asymptotics of  $\#\{x: f(x) = y \pmod{p^k}\}/p^{k(\dim X - \dim Y)}$ .

To prove our results, we use model theoretic tools (and in particular the theory of motivic integration, in the sense of uniform  $p$ -adic integration) to effectively study the collection  $\#\{x:f(x)=y \bmod p^k\}/p^{k(\dim X - \dim Y)}$ . If time allows, we will discuss these methods.

Based on a joint work with Raf Cluckers and Itay Glazer.

**Please Note the Unusual Place!**