Department of Mathematics, BGU

BGU Probability and Ergodic Theory (PET) seminar

On *Thursday, January ,4 2024*

At *11:10 – 12:00*

In *101-*

Liran Ron (BGU)

will talk about

Groups with Finitely Many Busemann Points

Abstract: Horofunction boundaries are a nice way to approach questions about the behavior of metric spaces at infinity and learn about their geodesics. In the case of Cayley graphs of finitely generated groups, they are also fruitful when studying group actions, algebraic properties and geometric properties (such as the growth rate of the group).

The basic construction is the embedding of the group G in a space of -1 Lipschitz functions on it, by the map sending x to the function b_x(y)=d(x,y)-d(x,1_G). This gives a compactfiication of G and a compact boundary. The elements in the boundary are called horofunctions. Some of the horofunctions (and in some cases, all of them) are realized as limits of geodesic rays in G, and these are called Busemann points.

The boundary depends on the metric on G, so dffierent Cayley graphs can give rise to dffierent (non-homeomorphic) boundaries. Thus, we are interested

in finding out which properties of the boundary are invariants of the group, and we are mainly focused on the cardinality in a broad sense (i.e. finite, countable or uncountable boundary) and the existence of a finite orbit under the group action on the boundary.

In this talk we will review quickly the main definitions and examples and then focus on groups with finitely many Busemann points. We will hopefully go through the main steps of proving that a group with finitely many Busemann points in every Cayley graph horofunction boundary are virtually-cyclic, and in that case every horofunction is a Busemann point.

Joint work with Ariel Yadin