Department of Mathematics, BGU

# BGU Probability and Ergodic Theory (PET) seminar

**On** *Thursday, December ,4 2025*

**At** *11:10 – 12:00*

**In** *101-*

Michael Lin

will talk about

## Entropy for power-bounded linear operators and Sarnak's conjecture

Abstract:

## Entropy for power-bounded linear operators and Sarnak's conjecture

**Michael Lin, Ben-Gurion University**

**Abstract** Let $\mu$ be the Möbius function, defined for $n \in \mathbb{N}$ by $\mu(1) = 1$, $\mu(n) = (-1)^n$ if $n$ is the product of $n$ distinct primes, and $\mu(n) = 0$ if $n$ has a square factor. This function is important in number theory, due to its connections to the Prime Number theorem and the Riemann Hypothesis.

Sarnak conjectured that whenver $X$ is a compact space and $\tau$ is a continuus map of $X$ into itself with zero toplogical entropy, then

$$\frac{1}{N}\sum_{k=1}^{N}\mu(k)f(\tau^k x) \to 0, \quad \forall f \in C(X),\ \forall x \in X.$$

When we denote $Tf = f \circ \tau$ for $f \in C(X)$, the above convergence is equivalent to

$$(1) \qquad \frac{1}{N}\sum_{k=1}^{N}\mu(k)T^k f \to 0 \ \text{ weakly}, \quad \forall f \in C(X).$$

It is our aim to study convergence like (1) for power-bounded operators on (real or complex) Banach spaces. A power-bounded $T$ on a Banach space $E$ becomes a contraction under the equivalent norm $|||x||| := \sup_{n\geq 0}\|T^n x\|$, so we assume $T$ to be a contraction.

For a contraction $T$ on $E$ we define an operator topological entropy $h^*_{top}(T)$, and prove that if Sarnak's conjecture is true, then $h^*_{top}(T) = 0$ implies

$$\frac{1}{N}\sum_{k=1}^{N}\mu(k)T^k v \to 0 \ \text{ weakly}, \quad \forall v \in E.$$

For some classes of operators $T$, in particular contractions in reflexive Banach spaces, we show $h^*_{top}(T) = 0$, and prove directly even the strong convergence $\|\frac{1}{N}\sum_{k=1}^{N}\mu(k)T^k v\| \to 0$ for every $v \in E$.

**Joint work with el Houcein el Abdalaoui (Rouen)**