

המחלקה למתמטיקה, בן-גוריון

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## קולוקוויום

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ביום שלישי, 28 ביוני, 2016

בשעה 14:30 – 15:30

ב-101 Math

ההרצאה

**$\mathbb{L}_p$  on contractions Ritt Positive**

חינתן על-ידי

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תקציר:

# Positive Ritt contractions of $L_p$

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Joint work with Guy Cohen and Christophe Cuny

Let  $P$  be a Markov operator on  $(\mathbb{S}, m)$  with  $m$   $\sigma$ -finite invariant. Then  $P$  extends to a contraction of each  $L_p(m)$ ,  $1 \leq p \leq \infty$ , and by the Hopf-Dunford-Schwartz theorem (1956),  $\frac{1}{n} \sum_{k=1}^n P^k f$  converges a.e. for  $f \in L_p(m)$ ,  $1 \leq p < \infty$ .

An important question in the theory of Markov operators is that of convergence of  $\{P^n f\}$ , in norm or a.e., for every  $f \in L_2(S, m)$ .

In 1961 E.M. Stein proved that if the Markov operator  $P$  is self-adjoint in  $L_2(m)$  with  $-1 \notin \sigma(P)$ , then  $\{P^n f\}$  converges a.e. for  $f \in L_p(m)$ ,  $1 < p < \infty$ ; however, convergence may fail for  $p = 1$ , even when  $m$  is finite. An important step in Stein's proof is to show that  $\sup_n n \|P^n(I - P)\|_2 < \infty$ .

Combining Stein's proof with Akcoglu's pointwise ergodic theorem (1975), we obtain a similar result for self-adjoint positive contractions in  $L_2$ .

A power-bounded  $T$  on a Banach space is called *Ritt* if  $\sup_n n \|T^n(I - T)\| < \infty$ .

Le Merdy and Xu (2012) studied Ritt contractions in *one*  $L_p$  space,  $1 < p < \infty$  fixed. They proved that if  $T$  is a positive Ritt contraction on  $L_p$ , then there is a  $(p, p)$ -strong maximal inequality, and for every  $f \in L_p(m)$  the sequence  $\{T^n f\}$  converges a.e.

In this talk we discuss some properties of Ritt contractions on complex Banach and Hilbert spaces, and exhibit several examples of positive Ritt contractions on  $L_p$  with applications of the Le Merdy-Xu convergence theorem.