Department of Mathematics, BGU

Operator Algebras

On *Thursday, June ,25 2020*

At *14:10 – 15:00*

In *Online*

Maria Gerasimova (Bar Ilan University)

will talk about

Isoperimetry, Littlewood functions, and unitarisability of groups

Abstract: See attached

Isoperimetry, Littlewood functions, and unitarisability of groups

Let Γ be a discrete group. A group Γ is called *unitarisable* if for any Hilbert space H and any uniformly bounded representation $\pi : \Gamma \to B(H)$ of Γ on H there exists a bounded operator $S : H \to H$ such that $S^{-1}\pi(g)S$ is a unitary representation for any $g \in \Gamma$. It is well known that amenable groups are unitarisable. It has been open ever since whether amenability characterises unitarisability of groups.

Dixmier: Are all unitarisable groups amenable?

One of the approaches to study unitarisability is related to the space of the Littlewood functions $T_1(\Gamma)$. We define the **Littlewood exponent** $\mathrm{Lit}(\Gamma)$ of a group Γ:

$$\mathrm{Lit}(\Gamma) = \inf \left\{ \mathrm{p} : \mathrm{T}_1(\Gamma) \subseteq \ell^{\mathrm{p}}(\Gamma) \right\}.$$

We will show that, on the one hand, $\mathrm{Lit}(\Gamma)$ is related to unitarisability and amenability and, on the other hand, it is related to some geometry of Γ.
We will discuss several applications of this connection. This is a joint work with Dominik Gruber, Nicolas Monod and Andreas Thom.

Isoperimetry, Littlewood functions, and unitarisability of groups

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