

Department of Mathematics, BGU

---

---

# Jerusalem - Be'er Sheva Algebraic Geometry Seminar

---

---

*On Wednesday, May ,26 2021*

*At 15:00 – 16:30*

**In**

Dominic Joyce (Oxford)

will talk about

## **Universal structures in enumerative invariant theories**

Abstract: In Gross-Joyce-Tanaka arXiv:2005.05637, we described a universal conjectural picture for enumerative invariants counting semistable objects in abelian categories/gauge theories, which claimed that under some assumptions: (i) one can construct invariants, as virtual classes in the rational homology of the “projective linear” moduli stack, for all topological invariants (fixed Chern classes etc), including classes with strictly semistables; (ii) these invariants satisfy a wall-crossing formula under change of stability condition, written in terms of a Lie bracket on the homology of the moduli stack, which came out of my project on vertex algebra structures on homology of moduli stacks. We proved the conjecture for representations of acyclic quivers. In work in progress, I have now proved/am proving versions of the conjectures for a broad family of settings

in Algebraic Geometry, in which invariants are formed using Behrend-Fantechi virtual classes. These include suitable quivers with relations, coherent sheaves on curves, surfaces and some 3-folds, and algebraic Seiberg-Witten invariants and Donaldson invariants of projective complex surfaces. The SW/Donaldson theory picture includes wall-crossing formulae, related to those of Mochizuki, which implicitly determine algebraic  $U(n)$  and  $SU(n)$  Donaldson invariants, of any rank, in terms of rank 1 Seiberg-Witten type invariants and invariants of Hilbert schemes of points, for any projective complex surface, without restriction on  $b^1$ , or  $b^2_+$ , or a simple type assumption. The talk will give an overview of this programme.

Lecture slides will be available temporarily from here: <https://www.dropbox.com/s/8nzw21zqwhrleg>