אוניברסיטת בן-גוריון בנגב

המחלקה למתמטיקה

סמסטר 2023–24–א

שם הקורס אלגברה קומוטטיבית

מספר קורס 201.2.2011

עמוד הקורס ברשת
https://sites.google.com/view/amyekut-math/home/teaching/comm-alg-2023-24-a

מרצה אחראי פרופ׳ אמנון יקותיאלי, <amyekut@bgu.ac.il>, חדר 202

שעות קבלה https://www.math.bgu.ac.il/he/teaching/hours

תקציר

here? goes what

דרישות והרכב ציון הקורס[1]

here? comes what

[1] דרישות הקורס יכולות להשתנות במהלך השבועיים הראשונים של הסמסטר, ויש לשים לב להודעות באתר הקורס

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פרופ' אמנון יקותיאלי
המחלקה למתמטיקה
אוניברסיטת בן גוריון
באר שבע 84105

13 September 2023

Graduate Course:

Commutative Algebra

Semester A (Fall) 2023-24

Broadcast on Zoom

General Description of the course. This is a second, advanced, course on commutative algebra. The first course (a prerequisite) was a preparation for the geometry of algebraic varieties over an algebraically closed field. The expectation is that the students are already familiar with noetherian rings, localization, the Hilbert Basis Theorem and Nullstellensatz, etc. (see topic 1 below). Our course will continue to new topics.

The first main topic of our course will be **completion in algebra**. We will study adic completion of rings and modules. For noetherian rings the Artin-Rees property is a key feature. We will also discuss completion in non-noetherian rings (which arises in arithmetic), completion of infinitely generated modules, and several related constructions. Of particular interest will be complete local rings.

The second main topic of the course is **differential commutative algebra**. We will learn about derivations and differentials. Then we will talk about étale ring homomorphisms, an extremely important yet difficult concept, unifying finite separable field extensions (from Galois theory), ramification (in number theory) and local diffeomorphisms (from differential geometry). We will approach étale homomorphisms from several directions, including the case of complete local rings. Some particular applications will be discussed (e.g. Azumaya algebras). If time allows we will also talk briefly about smooth ring homomorphisms.

Audience: The course is intended for graduate students at BGU. Strong undergraduate students, and *students from outside the BGU community*, are also welcome. All prospective students should contact the lecturer (me) by email before the course starts.

Catalogue no: 201.2.2011

Language: English.

Time and place: **Tuesday 12-14, broadcast on Zoom.** The first lecture is on 17 Oct 2023.

Prerequisite courses: "Introduction to Commutative Algebra" no. 201.1.7071 (or equivalent).

Organization: The course will consist of one weekly lecture (2 hours), presented on Zoom, and homework. The Zoom lecture recordings, and weekly typed notes, will be posted on the course web page.

attend without formal registration, but only with my permission.

Grades: For registered students only. The course grades are pass/fail.

Course web page:

https://sites.google.com/view/amyekut-math/home/teaching/comm-alg-2023-24-a

Course Topics: (depending on rate of progress.)

1. **Recalling basic commutative algebra.** Noetherian rings, Hilbert Basis Theorem and Nullstellensatz, localization of rings and modules, flatness, exact sequences, tensor products of modules and rings, infinite direct sums and products.
2. **Limits in algebra.** Direct and inverse limits of rings and modules, exactness of limits, examples.
3. **Adic completion**. Adic completion of rings and modules at ideals. Interpretation as metric completion.
4. **Completion of noetherian rings**. The Artin-Rees property, finitely generated modules, flatness. Completion at finitely generated ideals of non-noetherian rings, completion of infinitely generated modules. Examples from arithmetic.
5. **Complete local rings.** From valuations to completions, local fields, adeles of curves, singularities of curves. Hensel's Lemma. Cohen structure theorem.
6. **Differential commutative algebra**. Derivations and differential forms. Basic constructions, fundamental exact sequences. Relations with differential geometry (tangent bundles). Applications and examples.
7. **Étale ring homomorphisms**. The basic idea. Definitions. Relation to differential forms. Formally étale homomorphisms. Structural theorems. The case of complete local rings. Ramification in number theory. Some applications and examples. (This is a hard topic, so parts of it will only be outlined.)
8. **Smooth ring homomorphisms**. Survey only.

Bibliography.

1. Atiyah and MacDonald, "Introduction to Commutative Algebra", 1969.
2. Altman and Kleiman, "A Term of Commutative Algebra" (2021), free online book.
3. Eisenbud, "Commutative Algebra", GTM 150, Springer, 1995.
4. Matsumura, "Commutative Ring Theory", Cambridge, 1986.
5. Milne, "Étale Cohomology", Princeton, 1980.
6. A. Grothendieck and J. Dieudonné, "Éléments de géométrie algébrique" Book IV, Publ. IHES volumes 20 and 32, free at NUMDAM: Book IV part 1 and Book IV part 4.
7. A. Grothendieck, "Séminaire de Géométrie Algébrique" (SGA) Book 1, retyped edition (2004).
8. Stacks Project, an online mathematics resource, A.J. de Jong (ed).
9. Course notes, updated weekly, on the course web page.

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9 October 2023 (version 2)

Graduate Course:

Commutative Algebra

Semester A (Fall) 2023-24

Broadcast on Zoom

General Description of the course. This is a second, advanced, course on commutative algebra. The first course (a prerequisite) was a preparation for the geometry of algebraic varieties over an algebraically closed field. The expectation is that the students are already familiar with noetherian rings, localization, the Hilbert Basis Theorem and Nullstellensatz, etc. (see topic 1 below). Our course will continue to more advanced topics in commutative algebra, which are very important both for algebraic geometry and number theory.

The first main topic of our course will be **completion in algebra**. We will study adic completion of rings and modules. For noetherian rings the Artin-Rees property is a key feature. We will also discuss completion in non-noetherian rings (which arises in arithmetic), completion of infinitely generated modules, and several related constructions. Of particular interest will be complete local rings.

The second main topic of the course is **differential commutative algebra**. This topic is not included in most textbooks. We will learn about derivations and differentials. Then we will talk about étale ring homomorphisms, an extremely important yet difficult concept, unifying finite separable field extensions (from Galois theory), ramification (in number theory), and local diffeomorphisms (from differential geometry). We will approach étale homomorphisms from several directions, including in terms of complete local rings. Some particular applications will be discussed (e.g. Azumaya algebras). If time allows we will also talk briefly about smooth ring homomorphisms.

Audience: The course is intended for graduate students at BGU. Strong undergraduate students, and *students from outside the BGU community*, are also welcome. **All prospective students should contact me by email before the course starts**.

Catalogue no: 201.2.2011

Language: English.

Time and place: **Tuesday 12-14, broadcast on Zoom.** First lecture: TBA (due to ongoing war).

Prerequisite courses: "Introduction to Commutative Algebra" no. 201.1.7071 (or equivalent).

Organization: The course will consist of one weekly lecture (2 hours), presented on Zoom, and homework. The Zoom lecture recordings, and weekly typed notes, will be posted on the course web page.

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Course web page:

https://sites.google.com/view/amyekut-math/home/teaching/comm-alg-2023-24-a

Course Topics: (depending on rate of progress.)

1. **Recalling basic commutative algebra.** Noetherian rings, Hilbert Basis Theorem and Nullstellensatz, localization of rings and modules, flatness, exact sequences, tensor products of modules and rings, infinite direct sums and products.
2. **More basic commutative algebra**. Integral ring extensions. Structure of artinian rings.
3. **Limits in algebra.** Direct and inverse limits of rings and modules, exactness of limits, examples.
4. **Adic completion**. Adic completion of rings and modules at ideals. Interpretation as metric completion. Examples.
5. **Completion of noetherian rings**. The Artin-Rees property, finitely generated modules, flatness. Completion at finitely generated ideals of non-noetherian rings, completion of infinitely generated modules. Examples from arithmetic.
6. **Complete local rings.** From valuations to completions, local fields, adeles of curves, singularities of curves. Hensel's Lemma. Cohen structure theorem.
7. **Differential commutative algebra**. Derivations and differential forms. Basic constructions, fundamental exact sequences. Relations with differential geometry (tangent bundles). Applications and examples.
8. **Étale ring homomorphisms**. The basic idea. Definitions. Relation to differential forms. Formally étale homomorphisms. Structural theorems. The case of complete local rings. Ramification in number theory. Some applications and examples. (This is a hard topic, so parts of it will only be outlined.)
9. **Smooth ring homomorphisms**. Survey only.

Bibliography.

1. Atiyah and MacDonald, "Introduction to Commutative Algebra", 1969.
2. Altman and Kleiman, "A Term of Commutative Algebra" (2021), free online book.
3. Eisenbud, "Commutative Algebra", GTM 150, Springer, 1995.
4. Matsumura, "Commutative Ring Theory", Cambridge, 1986.
5. Milne, "Étale Cohomology", Princeton, 1980.
6. A. Grothendieck and J. Dieudonné, "Éléments de géométrie algébrique" Book IV, Publ. IHES volumes 20 and 32, free at NUMDAM: Book IV part 1 and Book IV part 4.
7. A. Grothendieck, "Séminaire de Géométrie Algébrique" (SGA) Book 1, retyped edition (2004).
8. Stacks Project, an online mathematics resource, A.J. de Jong (ed).
9. Course notes, updated weekly, on the course web page.

אוניברסיטת בן-גוריון בנגב

נושאי לימוד

רשימת נושאים

1. מודולים: מודולים חופשיים, סדרות מדוייקות, מכפלה טנזורית, מודולי הום, שטיחות.
2. אידיאלים ראשוניים ולוקליזציה: חוגים מקומיים, הלמה של נאקיאמה, הספקטרום של חוג, מימד וקשירות.
3. חוגים נתריאניים: משפט הבסיס של הילברט, הלמה של ארטין-ריס, השלמה, דירוג.
4. תורת המימד: משפט האפסים של הילברט, משפט הנירמול של נתר, מעלת טרנסצנדנטיות של שדות.