Ben Gurion University – Mathematics

Algebraic Geometry and Number Theory Seminar

Speaker **Amnon Yekutieli (BGU)**

Title **Weak Proregularity, Weak Stability, and the Noncommutative MGM Equivalence**

Date Wednesday, 9 November 2016

Time 15:10 – 16:30 (starts 15:10 sharp)

Location Room -101 in Building 58

Abstract

Let A be a commutative ring, and let \a := \frak{a} be a finitely generated ideal in it. It is known that a necessary and sufficient condition for the derived \a-torsion and the derived \a-adic completion functors to be nicely behaved is the weak proregularity of the ideal \a. In particular, the MGM Equivalence holds under this condition. Because weak proregularity is defined in terms of elements of the ring (specifically, it involves limits of Koszul complexes), it is not suitable for noncommutative ring theory.

Consider a torsion class T in the category M(A) of left modules over a ring A. We introduce a new condition on T: weak stability. Our first main theorem is that when A is commutative, an ideal \a in A is weakly proregular if and only if the corresponding torsion class T in M(A) is weakly stable.

It turns out that when the ring A is noncommutative, one must impose two more conditions on the torsion class T: quasi-compactness and finite dimensionality (these are new names for old conditions). We prove that for a torsion class T that is weakly stable, quasi-compact and finite dimensional, the right derived T-torsion functor is isomorphic to a left derived tensor functor. This result involves derived categories of bimodules. Some examples will be given.

The third main theorem is the Noncommutative MGM Equivalence, under the same assumptions on T. Finally, there is a theorem about derived left-sided and right-sided torsion for complexes of bimodules. This last theorem is a generalization of a result of Van den Bergh from 1997, and it corrects an error in a paper of Yekutieli-Zhang from 2003.

We expect that the approach outlined in this talk will open up the way to a useful theory of rigid dualizing complexes in the arithmetic noncommutative setting (namely without a base field).

The work above is joint with Rishi Vyas.

(updated 24 Oct 2016)