Topics in Group Theory (4 weekly hours)

Semester B, 2018

Yoav Segev

(see below in tiny fonts a bit about group theory)

(a) Permutation representation and the Sylow theorems.

(b) Representations of groups on groups, solvable groups, nilpotent groups, semidirect and central products.

(c) Permutation groups, the symmetric and alternating groups.

(d) The generalized Fitting subgroup of a finite group.

(e) p-groups.

(f) Extension of groups: The first and second cohomology and applications.

Bibliography

M. Aschbacher, *Finite Group Theory.*

M. Suzuki, *Group Theory, I, II.*

On Tuesday May 20, 2008, His Majesty King Harald presented the Abel Prize for 2008 to the two mathematicians, John Griggs Thompson of the USA and Jacques Tits of France. The ceremony took place in the University of Oslo Aula. The King was accompanied by Her Majesty Queen Sonja. The two prize winners will share the prize money, which amounts to NOK 6 million (1.2 million dollars).

Thompson and Tits have won the prize for their contributions to what is referred to as group theory. The Abel Committee Chairman, Professor Kristian Seip, said in his explanation that the prize is awarded "for their outstanding achievements in algebra and especially for their shaping of modern group theory".

Granting the Abel prize to two group theorist is not only an acknowledgment of their fantastic achievements but also of the central importance of group theory in mathematics. Indeed, the *classification of finite simple groups* is considered the greatest achievement in the mathematics of the 20th century.

Modern Group Theory views groups as *mathematical objects that reflect symmetry.* As Marcus Du Satouy puts it in his article for the Abel prize: "In the package that he (Galois the "father" of group theory that died in 1832 at the age of 20 in a dual) left after him was the beginning of a new language called group theory that would finally help mathematicians to articulate one of the most important concepts of nature: *symmetry* ..." "Symmetry is a central concept in both science and the arts. Symmetry is key to our evolutionary survival. It is an indicator of good genes. It controls the behavior of molecules, crystals and viruses. It has unlocked the secrets of the fundamental particles that make up the material world. It is also central to many of the codes that are used to preserve the integrity of data as it is transmitted around the world. Such error-correcting codes use symmetry to reveal when errors have crept into the data. For artists too symmetry is a central theme. From architecture to music, from poetry to painting, symmetry underpins many of the structures used in the creative world. But it wasn't until the nineteenth century that we finally had the language to answer the question: what is symmetry?"

There are many exciting problems yet to be investigated: problems intrinsic to the theory of groups; problems where group theory is applied to other branches of mathematics; and problems where group theory is applied outside of mathematics, in particular in modern physics and in computer science.