Joint and double coboundaries of transformations – an application of maximal spectral type of spectral measures

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Abstract

Let T be a bounded linear operator on a Banach space X; the elements of $(I-T)X$ are called *coboundaries*. For two commuting operators T and S, elements of $(I-T)X \cap (I-S)X$ are called *joint coboundaries*, and those of $(I-T)(I-S)X$ are *double coboundaries*. By commutativity, double coboundaries are joint ones. Are there any other?

Let θ and τ be commuting invertible measure preserving transformations of (Ω, Σ, m), with corresponding unitary operators induced on $L_2(m)$. We prove the existence of a joint coboundary $g \in (I-U)L_2 \cap (I-V)L_2$ which is not in $(I-U)(I-V)L_2$.

For the proof, let E be the spectral measure on $\mathbb{T}^2$ obtained by Stone's spectral theorem. Joint and double coboundaries are characterized using E, and properties of the maximal spectral type of E, together with a result of Foiaş on multiplicative spectral measures acting on L_2, are used to show the existence of the required function.