ERGODIC THEOREMS FOR CONVOLUTION POWERS

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ABSTRACT let G be a locally complact σ-compact group, with right Haar measure m, and let μ be a regular probability on G. The transition probability $P(t,A) := \mu(t^{-1}A)$ gives rise to the Markov operator

$$Pf(t) = P_\mu f(t) := \int_G f(ts)d\mu(s) = \mu * f(t).$$

Since (by Fubini) $\int_G Pf(t)dm(t) = \int_G f(t)dm(t)$, the right Haar measure m is invariant, and P is a contraction of $L_1(G,m)$ and of $L_\infty(G,m)$. The dual of P as a contraction on $L_1(m)$ is given by convolution (on L_∞) with the reflected probability $\check{\mu}(A) := \mu(A^{-1})$.

A bounded (measurable) function h is *invariant* if $\mu * h = h$ a.e.; it is called in our context *μ-harmonic*. If the only bounded harmonic functions are the constants, we call μ *ergodic*. A necessary condition for ergodicity is that μ be *adapted* – the closed subgroup generated by the support of μ is G. Clearly μ is adapted if and only if $\check{\mu}$ is adapted. An adapted probability need not be ergodic!

For two probabilities μ and ν we define their convolution by

$$\nu * \mu(A) = \int_G \int_G 1_A(ts)d\nu(t)d\mu(s),$$

and obtain the formula $P_\nu P_\mu = P_{\nu*\mu}$. Hence the powers of $P = P_\mu$ are given by $P_\mu^n = P_{\mu^{(n)}}$ (convolution powers).

From the general mean ergodic theorem, we have: $\|\frac{1}{n}\sum_{k=1}^n \mu^{(n)} * f\|_1 \to 0$ *for every $f \in L_1(G,m)$ with $\int_G f\,dm = 0$ if and only if $\check{\mu}$ is ergodic.*

In this talk we will discuss conditions on μ for the *complete mixing property*: $\|\mu^{(n)} * f\|_1 \to 0$ for every $f \in L_1(G,m)$ with $\int_G f\,dm = 0$.

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