OPERATOR ERGODIC THEOREMS

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ABSTRACT A bounded linear operator on a (real or complex) Banach space X is called *mean ergodic* if for every $x \in X$ the averages $A_n x := \frac{1}{n} \sum_{k=1}^{n} T^k x$ converge in norm for every $x \in X$ (with limit denoted EX).

I will present the first mean ergodic theorem due to Weyl (1909) for irrational rotations of the circle, von Neumann's theorem for unitary operators on Hilbert space (1932), Riesz's extension for contractions in Hilbert space (1936), and the proofs by Kakutani and Yosida (separately, 1938) for power-bounded operators on reflexive spaces.

Later developments which will be presented include uniform ergodic theorems (operator norm convergence of A_n), the Blum-Hanson theorem for convergence along all subsequences, weak mixing (convergence to zero of $\frac{1}{n} \sum_{k=1}^{n} |\langle x^*, T^k x - Ex \rangle|$ for every $x^* \in X^*$, $x \in X$), the Jacobs-deLeeuw-Glicksberg decomposition of reflexive complex Banach spaces spaces induced by unimodular eigenvalues, conditions for convergence of $T^n x$ in norm for every x.

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1