MARKOV OPERATORS

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ABSTRACT A *transition probability* on a measurable space (S, Σ) is a function $P : S \times \Sigma \longrightarrow [0, 1]$ satisfying:

(i) For fixed $s \in S$, $P(s, \cdot)$ is a probability on Σ.

(ii) For fixed $A \in \Sigma$, $P(\cdot, A)$ is measurable.

A transition probability defines the *Markov operator* $Pf(s) := \int f(t)P(s, dt)$ on bounded measurable functions, and the operator $\mu P(A) := \int P(s, A)d\mu(s)$ on finite signed measures on Σ. These are in duality: $\langle \mu P, f \rangle = \langle \mu, Pf \rangle$.

I will discuss some examples, explain how we obtain abstract Markov operators on $L_\infty(S, \Sigma, m)$, and describe some elements of the ergodic theory of L_1 contractions. The notions of ergodicity, weak mixing, mixing and complete mixing will be explained with relation to the general ergodic theorems, and the role of invariant probabilities $\mu P = \mu$ will be shown.

I will then describe the construction of the space of trajectories $(\Omega, \mathcal{B})$ and the Markov chain generated by P.

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