On Vietoris hyperspaces for some Boolean algebras

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We present joint results with Taras Banakh and Wieslaw Kubis. All notions will be defined in the lecture. This work is a continuation of "well-generated Boolean algebras", developed by Mati Rubin.

A Skula space X is a scattered compact 0-dimentional space with a partial order $\leq$ such that a subbase of clopen sets is the set of all $U_x := \{y \in X : y \geq x\}$ for $x \in X$.

On X we have two "cardinal invariants":
1. the (Cantor-Bendixson) height $ht(X)$ -corresponding to the ordinal for which the last derivative is nonempty finite derivative- and
2. the (well-founded) rank $rk(X)$ of X. Note that the rank is defined as the well-founded rank in $(X, \geq)$: e.g. rank 0 are maximal elements of X.

We define the Vietoris hyperspace $H(X)$ as follows. The elements are the closed and nonempty final substs of X. We endow $H(X)$ with the Vietoris topology, that is a subbase of clopen sets of $H(X)$ is the set $U^+ := \{F \in H(X) : F \subseteq U\}$ for every clopen final subset U of X.

Now if X is Skula then $H(X)$ is also Skula.

We show the relationships between Skula spaces, hyperspaces of Skula spaces, and the corresponding (Cantor-Bendixson) height and (well-founded) rank.

One example. To the Boolean algebra generated by an infinite almost disjoint family $\mathcal{A}$ on the set N of integers, we associate its space $K_{\mathcal{A}}$ as follows.
- Consider N as the set of maximal elements of $K_{\mathcal{A}}$.
- For each $A \in \mathcal{A}$ add a new element x_A and set $x_A < n$ iff $n \in A$. Denote by $D_{\mathcal{A}}$ the set of all x_A.
- Add a minimum ∞ to $N \cup D_{\mathcal{A}}$.

Consider $K_{\mathcal{A}} := \{\infty\} \cup D_{\mathcal{A}} \cup N$ as a Skula space. Then $K_{\mathcal{A}}$ is called a Mrówka space. In this example we have

(1) $ht(X) = rk(X) = 2$, and
(2) $ht(H(X)) = \omega$ and $rk(H(X)) = \omega + \omega$.