On classification of semigroups by algebraic, logical and topological tools

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October 23, 2022

Abstract

I plan to speak about the themes that I investigated last years:

- Identical inclusions (we consider a case of semigroups to save time).
- Topologies in languages and affine spaces
- Tarski's Finite Basis Problem
- Transformation semigroups identities
- Interrelations between Group Theory and Completely Simple Semigroups

Of course I can't explain any of these themes in details. Here I'll give only short presentation: explain my motivation, formulate few results, open problems and conjectures. Even this short presentation will be more detailed for the first themes and much less detailed for others. I'll be happy to answer your questions and later discuss in details any of these themes, problems, results and motivation if somebody will be interested in it. Here I'll give, for example, two theorems from the first theme.

Theorem 1 *Any semilattice is inclusively equivalent to one and only one of the following semilattices* $\{F_n, F_n^0, F_n^1, F_n^{0,1} | n \in N\}$*, where* F_n *is the free* n*-generated semilattice.*

Theorem 2 *Let* Φ *and* Ψ *are sets of identical inclusions;* S *and* T *are semilattices;* $\prec$ *is the lexicographic order on* $(\{a, b, c, d, e\}, \prec_1) \times ((N \cup \infty), \leq) \times (\{0, \alpha, \beta, \gamma, \delta, \epsilon\}, \prec_2) = C$*;* $C(\Phi), C(\Psi), C(S), C(T) \in C$*. Then*

- S *satisfies* Φ *if and only if* $C(S) \prec C(\Phi)$*;*
- $S \in \mathbf{I}(T)$ *if and only if* $C(S) \prec C(T)$*;*
- Φ *implies* Ψ *in in the class of semilattices if and only if* $C(\Phi) \prec C(\Psi)$*;*
- $\mathbf{I}(S) = \mathbf{I}(\Phi) \Leftrightarrow C(S) = C(\Phi)$*;*
- $\mathbf{I}(S) = \mathbf{I}(T) \Leftrightarrow C(S) = C(T)$*;*
- Φ *is equivalent to* Ψ *in the class of semilattices* $\Leftrightarrow C(\Phi) = C(\Psi)$*.*