Non-classifiability of ergodic flows up to time change

Dating back to the foundational paper by John von Neumann, a fundamental theme in ergodic theory is the *isomorphism problem* to classify invertible measure-preserving transformations (MPTs) up to isomorphism. In a series of papers, Matthew Foreman, Daniel Rudolph and Benjamin Weiss have shown in a rigorous way that such a classification is impossible. Besides isomorphism, Kakutani equivalence is the best known and most natural equivalence relation on ergodic MPTs for which the classification problem can be considered. For ergodic flows $\{S_t\}_{t\in\mathbb{R}}$ and $\{T_t\}_{t\in\mathbb{R}}$, Kakutani showed that the two flows have Kakutani equivalent transformations as cross-sections if and only if the flows are isomorphic up to a time change. Here, a time change of a flow is a reparametrization of the orbits of the flow such that each orbit is mapped to itself by an orientation-preserving homeomorphism of the parameter space.

In joint work with Marlies Gerber we prove that the Kakutani equivalence relation of ergodic MPTs is not a Borel set. This shows in a precise way that the problem of classifying such transformations up to Kakutani equivalence is also intractable. In particular, our results imply the non-classifiability of ergodic flows up to isomorphism after a time change.