Non-commutative measures and Non-commutative Function Theory in the unit row-ball

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In this talk we extend the classical correspondence between finite, positive and regular 'Clark' measures on the complex unit circle and contractive analytic functions in the complex unit disk from one to several non-commuting variables. In this non-commutative (NC) theory, NC measures are defined as positive linear functionals on a certain self-adjoint subspace of the *Cuntz–Toeplitz algebra*, the C^*-algebra generated by the left creation operators on the full Fock space, or *free Hardy space* of square–summable power series in several NC variables. This is a natural generalization of the classical Hardy space of square–summable Taylor series in the complex unit disk and we will develop exact analogues of classical results on the interplay between Hardy Space Theory in the disk and Measure Theory on the circle. Elements of the free Hardy space can be viewed as analytic free non-commutative functions, in the sense of Kaliuzhnyĭ-Verbovetskyĭ and Vinnikov, acting on an *NC unit row-ball* of $d-$tuples of matrices of all finite sizes.

Our results include an NC Fatou Theorem for the 'Radon–Nikodym derivative' of any positive NC measure with respect to a canonical NC Lebesgue measure, an NC Lebesgue decomposition and a characterization of the NC Clark measures of contractive NC rational multipliers of the free Hardy space.