Redundant generation of groups acting on trees

Gaussian elimination iteratively applies row operations to move from some n-tuple S generating an m-dimensional vector subspace $V \subseteq \mathbb{F}^k$ $(n > m)$ to another generating tuple, $S^{'}$, which contains m distinguished vectors that are guaranteed to generate V. Thus one can always read the dimension off of a row-equivalent generating tuple, regardless of the starting point. It is natural to generalize this from vector spaces to groups: given a generating set S of size n for a group G with $rank\,(G) = m$, $n > m$, can one apply Nielsen transformations to arrive at a generating set $S^{'}$ which admits a generating proper subset of size m? A generating set S for which we can do so is called redundant. For $G = \mathrm{PSL}_2\,(\mathbb{Q}_p)$ (which acts on its Bruhat-Tits tree), we prove redundancy for large classes of generating sets. In order to do so we will characterize the obstacles to S generating a dense subgroup of G, describe "typical" dense subgroups, and demonstrate a method to find such subgroups via Nielsen transformations.