Positive Ritt contractions of L_p

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Joint work with Guy Cohen and Christophe Cuny

Let P be a Markov operator on $(\mathbb{S}, m)$ with m σ-finite invariant. Then P extends to a contraction of each $L_p(m)$, $1 \le p \le \infty$, and by the Hopf-Dunford-Schwartz theorem (1956), $\frac{1}{n}\sum_{k=1}^{n} P^k f$ converges a.e. for $f \in L_p(m)$, $1 \le p < \infty$.

An important question in the theory of Markov operators is that of convergence of $\{P^n f\}$, in norm or a.e., for every $f \in L_2(S, m)$.

In 1961 E.M. Stein proved that if the Markov operator P is self-adjoint in $L_2(m)$ with $-1 \notin \sigma(P)$, then $\{P^n f\}$ converges a.e. for $f \in L_p(m)$, $1 < p < \infty$; however, convergence may fail for $p = 1$, even when m is finite. An important step in Stein's proof is to show that $\sup_n n\|P^n(I-P)\|_2 < \infty$.

Combining Stein's proof with Akcoglu's pointwise ergodic theorem (1975), we obtain a similar result for self-adjoint positive contractions in L_2.

A power-bounded T on a Banach space is called *Ritt* if $\sup_n n\|T^n(I-T)\| < \infty$.

Le Merdy and Xu (2012) studied Ritt contractions in *one* L_p space, $1 < p < \infty$ fixed. They proved that if T is a positive Ritt contraction on L_p, then there is a (p,p)-strong maximal inequality, and for every $f \in L_p(m)$ the sequence $\{T^n f\}$ converges a.e.

In this talk we discuss some properties of Ritt contractions on complex Banach and Hilbert spaces, and exhibit several examples of positive Ritt contractions on L_p with applications of the Le Merdy-Xu convergence theorem.