Title: *Galois groups of local fields, Lie algebras and ramification*

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Abstract. Let $p > 2$ be a prime number. By local field K we mean a finite field extension of either the field of p-adic numbers $\mathbb{Q}_p$ (the mixed characteristic case), or the field of formal Laurent series $\mathbb{F}_p((X))$ (the characteristic p case).

If Γ_K is the absolute Galois group of K then its maximal p-quotient $\Gamma_K(p)$ contains the major information about Γ_K. If a primitive p-th root of unity $\zeta_p \notin K$ then $\Gamma_K(p)$ is a free pro-p-group, if $\zeta_p \in K$ then $\Gamma_K(p)$ has finitely many generators and one relation (the Demushkin relation) similar to the relation in the fundamental group of Riemannian surface of some genus g. These results were obtained in 1960's but did not find substantial applications because $\Gamma_K(p)$ appeared to be very weak invariant of K.

On the other hand, a local class field theory (created also in 1960's) gave a perfect tool to describe the maximal abelian quotient Γ_K^{ab} (and $\Gamma_K(p)^{ab}$) together with its functorial behaviour with respect to K.

Since that time there were real expectations for creating so-called nilpotent class field theory (an analog of class field theory for field extensions with nilpotent Galois groups) but it seems we are still very far from a functorial approach to this problem.

In 1990's the author suggested new techniques, to study p-extensions of local fields with Galois groups of nilpotent class $< p$. This technique allows us to fix identification of the maximal quotient $\Gamma_{<p}$ of $\Gamma_K(p)$ of nilpotent class $< p$ with the group obtained from some Lie algebra via the Campbell-Hausdorff composition law. We described explicitly $\Gamma_{<p}$ together with its ramification subgroups $\Gamma_{<p}^{(v)}$, $v \geqslant 0$, and this construction gives us a very strict invariant of K. Earlier, these methods required highly non-trivial calculations with Lie algebras and worked only in the characteristic p case. Recently, we developed another approach based on the theory of deformations and applied it to the mixed characteristic case as well. In particular, we obtained explicit form of the Demushkin relation in terms of ramification subgroups of $\Gamma_{<p}$.