BEN GURION UNIVERSITY **אוניברסיטת בן גוריון בנגב**

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פרופ' אמנון יקותיאלי
המחלקה למתמטיקה
אוניברסיטת בן גוריון
באר שבע 84105

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Algebraic Geometry – Schemes 2

BGU, Spring 2020-21

Catalogue no. 201.2.2121

General Information. The course will be in English. It is a continuation of "Algebraic Geometry – Schemes 1" from the fall semester.

The course web page (with up-to-date information):
https://www.math.bgu.ac.il/~amyekut/teaching/2020-21/schemes-2/course_page.html

Participants from outside the BGU community (and from all over the world) are welcome. Please distribute this page to interested participants.

See the "first day handout" on the course page for technical stuff.

Prerequisites and Level. The course is intended for graduate students and advanced undergraduate students.

Participants of the course should have good knowledge of these topics: the material of the course "Schemes 1" (i.e. categories and functors, sheaves, ringed spaces and their maps and sheaf operations); some classical algebraic geometry (varieties over an algebraically closed field, or at least algebraic curves); commutative algebra; and some Galois theory. It will be very useful to also know some homological algebra, algebraic topology, and the geometry of real differentiable or complex analytic manifolds.

The level of the course course will be calibrated – in terms of rate of progress and sophistication of the presentation – to the audience. I hope that we can go faster than in the first semester (since gaps in preparation have been filled).

Please send me an email if you are considering attending the course, indicating which of the topics above you have learned (in a formal course or privately), your academic status (degree, year and institution), and whether you intend to register or just to listen.

Course Topics : (tentative)

1. **Locally Ringed Spaces.** Recalling ringed spaces, sheaves of modules, maps of ringed spaces and sheaf operations. Locally ringed spaces and their maps. Examples from differential and classical algebraic geometry. Locally free sheaves, vector bundles, Picard group.
2. **Affine Schemes**. Definitions and basic properties. Examples from classical algebraic geometry and

from arithmetic. Noetherian affine schemes. Coherent and quasi-coherent sheaves. Maps of affine schemes, closed subschemes, principal open subschemes.

3. **Schemes and their Maps**. Definitions and basic properties. Noetherian schemes. Examples from classical algebraic geometry. Coherent and quasi-coherent sheaves. Fiber products and base change. Finite, finite type, flat, separated and proper maps of schemes.
4. **Projective Spaces.** Definition of the projective space $\mathbf{P}^n_{\mathbb{K}}$. Line bundles and maps to projective space. Proving that projective maps are proper. Computing the Picard group of $\mathbf{P}^n_{\mathbb{K}}$. Blowups and some birational geometry.
5. **Sheaf Cohomology.** Nonabelian 1-st Čech cohomology, abelian sheaf cohomology (Čech and derived functor), Serre Duality, Riemann-Roch for curves, etc.
6. **Algebraic Differential Calculus**. Smooth and étale maps, differential forms, differential operators, etc.
7. **The Functor of Points and Moduli Spaces**. Yoneda embedding. Examples of moduli spaces. A first glance at algebraic spaces and algebraic stacks.
8. **Group Schemes and their Lie Algebras**. Definitions and basic properties. Examples of affine group schemes (tori, $\mathrm{GL}_{n,\mathbb{K}}$), abelian varieties. The Lie algebra functor.

Bibliography:

1. Course lecture notes (to be posted weekly).
2. Hartshorne, "Algebraic Geometry", Springer.
3. Matsumura, Commutative Ring Theory", Cambridge U. Press, 1989.
4. Altman and Kleiman, "A Term of Commutative Algebra" (free online book)
5. Kashiwara and Schapira, "Sheaves on Manifolds", Springer.
6. de Jong (Ed.), "The Stacks Project", https://stacks.math.columbia.edu.
7. Olsson, "Algebraic Spaces and Stacks", AMS.
8. Eisenbud and Harris, "The Geometry of Schemes", Springer.