Sarnak's Möbius disjointness conjecture for dendrites and Veech systems

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Abstract. A dendrite is a connected compact metric space X for which for any two distinct points u, v there exist a point w in X such that one can write $X \setminus \{w\}$ as union of U and V with U and V are open disjoint neighborhoods of u and v respectively. For any point x of X, we defined its *order* as the number of connected component of $X \setminus \{x\}$. If the order is one then the point is called endpoint. The dendrite is called Gehman dendrite if the set of its endpoints is closed. A dendrite system is a flow (X, T), where X is a dendrite and T is a continuous map on it.

The Veech system is a bitopological space for which the strong topology is metrizable and separable and the map on it is an isometry with respect to the metric.

In this setting, we consider Sarnak's Möbius disjointness conjecture. This conjecture assert that all dynamical systems with zero topological entropy satisfy the so-called Möbius randomness law. This later law presume that the Möbius function changes sign randomly. It turns out that Sarnak's conjecture is related to Chowla's conjecture which predict that the Liouville function is normal.

We recall that the Liouville function assigns the value $+1$ to n if the number of prime factors of n, counted with multiplicities, is even and -1 if not. The Möbius function coincide with the Liouville function on the set of square-free number and assigns the value zero otherwise. We recall that the number n is not square-free if there is a prime number p such that p^2 divides n.

In my talk, I will present my recent joint work with in which G.hassen Askri and H. Marzougui (University of Carthage, Tunisia) we establish that the Möbius disjointness conjecture holds for some class of Gehman dendrites. I will further present my very recent joint work with M. Neruraker (Rutgers university, USA) in which we establish that this conjecture holds also for Veech systems and systems with singular spectra. As a consequence, we obtain some kind of improvement of Motohashi-Ramachandra result on the Mertens function in short interval.

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