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★**Derived categories.**

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Yekutieli's text on derived categories provides a substantive reference on the fundamentals of this topic, delving into the intricacies of many tools needed for significant work in this area. Derived categories were born, in part, of the desire to provide an appropriate setting for the derived functors of classic homological algebra. Let M be an abelian category, such as the category of modules over a ring. The category of complexes over M contains, for example, all of the usual resolutions from classic homological algebra. However, one would often like certain morphisms in this category—namely, the morphisms whose cohomology is an isomorphism—to be invertible. So, just as one localizes a ring, a categorical localization at these morphisms produces a new category—the derived category—which is in many ways a more adequate setting for derived functors. Since their introduction around 1960 by A. Grothendieck and J.-L. Verdier, and first publication in R. Hartshorne's book *Residues and duality* [Lecture Notes in Mathematics, 20, Springer, Berlin, 1966; MR0222093], derived categories have become an important tool and ubiquitous throughout algebra. However, even today they are often viewed as somewhat mysterious. An underlying goal in this book, to quote the author, "is to demonstrate that the theory of derived categories is difficult, but not mysterious".

This text is intended as a self-contained and thorough approach to the fundamentals of derived categories and their tools, as well as a selection of applications of the theory. A reader already familiar with rings and categories will find—perhaps after a glance at Chapters 1 and 2—that Chapters 3 through 7 provide the background needed to understand what derived categories are. On the other hand, in order to understand the tools required for actually working in derived categories, one will need to spend significant time with Chapters 8 through 12; these chapters include detailed accounts of derived functors and how to work with them from both an abstract and a concrete perspective. This first portion of the text (Chapters 1 through 12) is mostly based on classic literature but has the goal of collecting the theory of derived categories as a single progression of ideas as well as putting it into the differential graded (DG) setting. The remaining chapters (Chapters 13 through 18) include topics such as dualizing complexes and applications of derived categories to commutative and noncommutative algebra.

One novel feature of this text is that most of the theory is developed in the following extension of the classic setting. Let A be a DG ring and M an abelian category, both over a commutative base ring $\mathbb{K}$. A *DG A-module in* M is a complex X of objects in M together with a DG ring homomorphism $A \to \mathrm{End}_{\mathsf{M}}(X)$. The *DG category of DG A-modules in* M is denoted $\mathbf{C}(A, \mathsf{M})$; its morphism sets are DG $\mathbb{K}$-modules. This includes the usual notion of the category of complexes over an abelian category, but allows for extra flexibility in terms of DG structure. The category $\mathbf{C}(A, \mathsf{M})$ has an associated homotopy category, denoted $\mathbf{K}(A, \mathsf{M})$, and localization at the quasi-isomorphisms then produces the *derived category of DG A-modules in* M, denoted $\mathbf{D}(A, \mathsf{M})$.

Individuals hoping to learn about derived categories from the ground up—and willing to commit a significant amount of time to the process—will find that this book provides a solid foundation for the topic. Researchers already familiar with some of the theory may benefit from reading this linear development of derived categories, as it also offers a number of enlightening historical and contextual remarks along the way. For a group of more advanced students—such as those already quite comfortable with rings and categorical methods—it is possible to envision a one- or two-semester course, coming from carefully selected portions of Chapters 3 through 12, that introduces derived categories and their tools.

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