Princeton, le 24 février 2016

Dear Yekutieli,

On second thoughts, the way you formulate things with two groups $H \to G$ seems artificial to me. As I understand, the guiding intuition is that on a disk $D$, a monodromy around $\partial D$ is some kind of integral of the curvature inside, and one wants to define this kind of integral, the basic picture being

Here is a setting I prefer, even if it is not more general than yours. Instead of $\underset{\beta}{H} \qquad \underset{\alpha}{G}$, I want

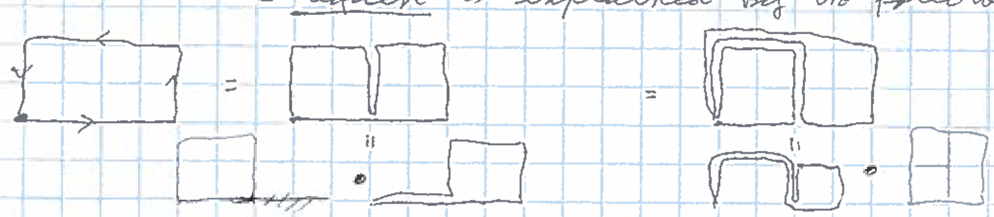
1) a bundle in groups $\underline{H}$

2) a connection on this bundle, compatible with the group structure. Let $R$ be the curvature of the induced connection on Lie

3) a 2-form $\beta$ with values in the bundle of Lie algebras Lie $\underline{H}$. <u>Required</u> : $\mathrm{ad}\,\beta = R$

For $o$ a base point, and for an homotopy to $o$ of a loop based at $o$, one wants a multiplicative integral $\int \beta$ with values in the fiber $\underline{H}_o$ of $\underline{H}$ at $o$.

The <u>request</u> is explained by the following picture



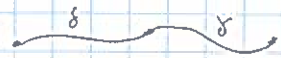
This is not more general than what you do, but has clearer (for me) invariance properties.

Not more general : because of the connection, all fibers of $\underline{H}$ are isomorphic. Fix $H$ isomorphic to all fibers. Then, for $G = \underline{\mathrm{Aut}}(H)$, the data of $\underline{H}$ amounts to that of the $G$-torsor $\left( \mathrm{Isom}(H, \underline{H}_t) \text{ at } t \right)$, and the connection on $\underline{H}$ to a connection on that torsor. As the story takes place in a contractible setting, one can even fix an isomorphism of $\underline{H}$ with the constant bundle with fiber $H$. This trivializes the torsor and turns the connection into a 1-form ($\alpha$) with values in Lie $G$.

Conversely, your $\begin{matrix} H & G \\ \beta & \alpha \end{matrix}$ gives, by $\alpha$ and the action of $G$ on $H$, a connection on the constant bundle of groups with fiber $H$.

The curvature case is that if $Q$ is a $H$-torsor with connection, twisting $H$ by $Q$ (for the inner action of $H$ on itself) we obtain $\underline{H}$, with connection, acting on $Q$, the curvature of $Q$ is in Lie $\underline{H}$, and by "integration", the curvature gives monodromy. For vector bundles ($\leftrightarrow GL(n)$-torsor), $(V, \nabla)$ gives a bundle of groups with connection $GL(V)$, the curvature is with values in Lie $GL(V) = \underline{\mathrm{End}}(V)$, and "$\int$" curvature = monodromy

Signs : I define composition of paths as

$\gamma\delta =$ 

to have parallel transport acting on the left. This presumably means I want to define 1-dim multiplicative integration as  : $\int = g_n \cdots g_1$

($g_i$ = $\underbrace{\text{infinitesimal element}}_{=\text{element of Lie}}$ $\beta(\Delta t)$ of the group ).

This may lead to inessential discrepancies with your notations.

Best

P. [signature]