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Derived Categories III

Course in 1st Semester 2016-17

Audience & Prerequisites. This is an advanced course, aimed at M.Sc. and Ph.D. students, post-docs and researchers. *Participants from outside the BGU community are welcome.* The lectures will be in English.

The course is a continuation of *Derived Categories I and II* that were given in the academic year 2015-16. It is expected that participants shall have a reasonable knowledge of the material covered in the first two courses (see below). A familiarity with commutative algebra, ring theory, algebraic geometry and algebraic topology is also recommended.

Organization. The course will meet once a week for a 2 hour lecture.
Time: Wednesday 12:00-14:00
Location: building 58 room -101
First Meeting: 2 November 2016

Potential participants are urged to get in touch with the lecturer in advance. The course notes will be published on the arxiv, and very likely also as a book. Here is a link to the course web page:
http://www.math.bgu.ac.il/~amyekut/teaching/2016-17/der-cats-III/course_page.html

On the subject. See the prevoius announcements on the course web page.

Content of the first two courses. We started with a review of categories (in general and abelian). After that we made an in-depth study of DG (differential graded) algebra: DG rings, DG modules, DG categories and DG functors. We introduced the DG category $\mathsf{C}(A, \mathsf{M})$ of DG A-modules in M, where A is a DG ring and M is an abelian category. This new framework includes in it both the category of unbounded complexes in M and the category of DG A-modules.

The next topic was pretriangulated categories and triangulated functors. We proved that the homotopy category $\mathsf{K}(A, \mathsf{M})$ of $\mathsf{C}(A, \mathsf{M})$ is pretriangulated. We also proved that for any DG functor $F : \mathsf{C}(A, \mathsf{M}) \to \mathsf{C}(B, \mathsf{N})$, the induced functor $F : \mathsf{K}(A, \mathsf{M}) \to \mathsf{K}(B, \mathsf{N})$ is triangulated.

We then made a thorough study of localization of categories. The localization of $\mathsf{K}(A, \mathsf{M})$ with respect to the quasi-isomorphisms is the derived category $\mathsf{D}(A, \mathsf{M})$. The category $\mathsf{D}(A, \mathsf{M})$

is pretriangulated, and the localization functor $\mathrm{Q} : \mathsf{K}(A, \mathsf{M}) \to \mathsf{D}(A, \mathsf{M})$ is both a triangulated functor and an Ore localization.

Next we talked about derived functors. To enable a precise treatment of this concept, we introduced some 2-categorical language. We proved the uniqueness of the left and right derived functors $\mathrm{L}F, \mathrm{R}F : \mathsf{D}(A, \mathsf{M}) \to \mathsf{E}$ into any pretriangulated category E.

Existence of derived functors relies on the availability of resolutions. We defined K-projective, K-injective and K-flat resolutions in $\mathsf{K}(A, \mathsf{M})$. Then we proved existence of these resolutions in several important algebraic and geometric situations.

The notes from parts I-II of the course (in an unedited version) are available on the course web page.

Topics for the third course. Here is a tentative list of topics – the actual choice of topics will be influenced by the participants. Some of the material will be taken from textbooks, and some from research papers. There will be a few guest lectures.

(1) **Derived categories in commutative algebra:** dualizing complexes, affine Grothendieck duality, local duality, rigid dualizing complexes.
(2) **Derived categories in algebraic geometry:** derived direct and inverse image functors, rigid residue complexes, global Grothendieck duality, applications to birational geometry (survey), perverse sheaves (survey), l-adic cohomology and Poicaré-Verdier duality (survey).
(3) **Derived categories in noncommutative ring theory:** dualizing complexes, tilting complexes, the derived Picard group, derived Morita theory.
(4) **Derived algebraic geometry (survey):** nonlinear derived categories, infinity categories, derived algebraic stacks, applications to mathematical physics.