Course Notes:

Homological Algebra

BGU, Spring 2019-20

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Available here:

https://www.math.bgu.ac.il/~amyekut/teaching/2019-20/homol-alg/course_page.html

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0. Introduction

comment: Start of Lecture 1, 11 March 2020

This course is a continuation of the course "Commutative Algebra" from the previous semester. We will use the notes [Yek1] a lot.

Here is a revised syllabus for this course. It is tentative: I will change material, and the order of presentation, as I go along.

Course Topics:

(1) Adjoint functors, equivalences and exactness.
(2) Bimodules and noncommutative tensor products.
(3) Projective modules, invertible bimodules and Morita Theory.
(4) Injective modules, including Matlis Theory for noetherian commutative rings.
(5) Complexes of modules, homotopies and homotopy equivalences, quasi-isomorphisms.
(6) The long exact cohomology sequence.
(7) Projective, flat and injective resolutions of modules.
(8) Left and right derived functors.
(9) Applications of derived functors to commutative algebra.
(10) Further applications of derived functors and cohomology, including non-abelian cohomology.

The notes from the course "Homological Algebra" [Yek2] from two years ago cover topics 5-8 roughly. (They also include the material on categories and functors, that we already covered in [Yek1].)

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[Yek2] A. Yekutieli, Course Notes: Homological Algebra (2017-18), https://www.math.bgu.ac.il/~amyekut/teaching/2017-18/hom-alg/course_page.html.

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