Geometric Topology and Geometry of Banach Spaces

Abstracts of Plenary Lectures

The space $\mathcal{L}(\mathfrak{X})$ for certain Banach spaces

Spiros Argyros
National Technical University of Athens, Greece

A Banach space $\mathfrak{X}$ satisfies the " scalar-plus-compact" property if every bounded linear operator $T \in \mathcal{L}(\mathfrak{X})$ is of the form $\lambda I + K$ with K a compact operator in $\mathcal{L}(\mathfrak{X})$. The space $\mathfrak{X}$ satisfies the "invariant subspace property" if every $T \in \mathcal{L}(\mathfrak{X})$ has a non trivial closed invariant subspace. In this talk we will present a variety of $\mathcal{L}_\infty$ spaces with the "scalar-plus-compact" property and we will explain the significant role of the $\mathcal{L}_\infty$ structure to achieve such a property. In the second part we will discuss some new spaces with non separable $\mathcal{L}(\mathfrak{X})$ satisfying the "invariant subspace property". At the end we will introduce the "forward bounded transfer" and "backward bounded transfer" and we will present some research related to those concepts.

On complements of compact sets in the Hilbert cube

Alexander Dranishnikov
University of Florida, USA

We define a cohomological version of weakly infinite dimensional compacta (cWID) for coefficients in a ring. The main result is that for any coefficient ring R the complement of cWID compactum in the Hilbert cube is R-acyclic. We show for any ring R that the class of cWID compacta contains all weakly infinite dimensional compacta (WID) and all compacta with finite cohomological dimension with respect to R.

Variants of decomposition complexity in coarse geometry

Jerzy Dydak
University of Tennessee, USA

By decomposition complexity I mean any coarse invariant defined in terms of splitting, for any positive number $r > 0$, a metric space X into subsets that can be expressed as a union of a uniformly bounded family that consists of r-disjoint sets. The strongest such invariant is asymptotic dimension of Gromov, the weakest such invariant is an analog of weak paracompactness. Up to now there were a few intermediate invariants: finite decomposition complexity (Yu and collaborators), straight finite decomposition complexity (Dranishnikov and Zarichnyi), and Asymptotic Property C (Dranishnikov). Recently, Yamauchi gave a very ingenious proof that infinite direct sum of integers has Asymptotic Property C and T. Davila generalized it by showing that any infinite direct product of groups of finite asymptotic dimension has Asymptotic Property C. I will outline how to define Asymptotic Property D, a coarse invariant stronger than Asymptotic Property C. By using ideas from Matrix Algebra, I will show that any infinite product of spaces of finite asymptotic dimension has Asymptotic Property D, thus generalizing theorems of Yamauchi and Davila.

Topology of large random spaces

Michael Farber
Queen Mary's College of London, Great Britain

I will discuss probabilistic models generating random simplicial complexes. One is able to predict their topological properties with probability tending to one when the spaces are large, i.e. depend on a growing number of independent random variables.

An infinite-dimensional phenomenon in finite-dimensional topology

Steve Ferry
Rutgers University, USA

Given a monotone continuous function $r : [0, R) \to [0, \infty)$ with $r(0) = 0, r(t) \geq t$, we study the class of closed n-dimensional topological manifolds M such that each ball of radius $t < R$ in M contracts to a point in the concentric ball of radius $r(t)$. We think of r as a continuous analog of feature size. A collection C of such manifolds has a covering function p if for each $\epsilon > 0$ and M in C, M has a $p(\epsilon)$-element open cover by sets of diameter $< \epsilon$.

If the elements of C have a uniform diameter bound D and $n > 6$, C contains only finitely many diffeomorphism types. However, it is not the case that any two manifolds from C which are sufficiently close in the Gromov-Hausdorff sense must be homeomorphic. This has applications to the problem of reconstructing a manifold from a finite sample.
Coauthors: A. Dranishnikov, S. Weinberger.

On operator ranges

Vladimir Fonf
Ben-Gurion University of the Negev, Israel

This is a joint work with S. Lajara, S. Troyanski, and C. Zanco.

Given bounded linear operators $A : X \to E$ and $T : E \to Y$ between Banach spaces, with E separable, and a subspace $L \subset E$ such that $L \cap AX = \{0\}$, we provide sufficient conditions to ensure the existence of an infinite-codimensional subspace $L_1 \subset E$ such that $L \subset L_1$, $L_1 \cap AX = \{0\}$ and $\operatorname{cl} TL_1 = \operatorname{cl} TE$. Some applications to the study of quasi-complemented subspaces of a Banach space are also given.

"Irrational" Convexity

Vitali Milman
Tel Aviv University, Israel

Do we have enough examples of Convex Bodies? Is diversity of our standard examples enough to understand Convexity? In the talk we demonstrate many different constructions which are analogous to constructions of irrational numbers from rationals. We show, following Il.Molchanov, that the solutions of "quadratic" equations like $Z^o = Z + K$ always exists (where Z^o is the polar body of Z; Z and K are convex compact bodies containing 0 in the interior). Then we show how the geometric mean may be defined for any convex compact bodies K and T (containing 0 into their interior). We also construct K^a for any centrally symmetric K and $0 < a < 1$, and also Log$K$ for K containing the euclidean ball D (and $K = -K$). Note, the power a cannot be above 1 in the definition of power! All these constructions may be considered also for the infinite dimensional setting, but this is outside the subject of the talk.

These results are joint with Liran Rotem.

Obstructions to embeddability of metric spaces in L_p spaces

Gideon Schechtman
Weizmann Institute of Science, Israel

I shall present a few inequalities on metric spaces which serve as obstructions to good Lipschitz (and uniform) embeddings of certain metric spaces into L_p spaces. Most of the recent results are due to Assaf Naor and myself. One such inequality, The metric X_p inequaity, completes the search for bi-Lipschitz invariants that serve as an obstruction to the embeddability of L_p spaces into each other. Another, more recent one, The metric property α inequality, serves as an obstruction to the embeddability of Schatten classes (and natural discrete subsets of them) into L_p spaces.

Abstracts of Regular Talks

Second Countable Approximation of Locally Compact Groupoids and an Application to Disintegration Theorems for Fell Bundles

Kyle Austin
Ben-Gurion University of the Negev, Israel

The representation theory for locally compact groupoids has been extensively developed for second countable groupoids. In this talk, I will explain how my collaborators and I have devised a method of approximating sigma compact (equivalently Lindelof) locally compact groupoids by second countable ones. I will explain how our approximations are compatible with twists and with Haar systems. As an application, I will show how we have extended the Muhly-Williams disintegration theorem for representations of the algebra of sections of Fell Bundles from second countable groupoids to sigma compact groupoids. The reader may ask why all this matters. The reason all of this matters is that there are natural locally compact groupoids that are not second countable that are of extreme importance in geometric group theory; namely, the coarse groupoid of a coarse space.

Topological spaces with an ω^ω-base

Taras Banakh
University in Kielce, Poland and University of Lviv, Ukraine

Given a partially ordered set P, we say that a topological space X has a *local P-base* if each point $x \in X$ has a neighborhood base $(U_\alpha[x])_{\alpha\in P}$ such that $U_\beta[x] \subset U_\alpha[x]$ for all $\alpha \le \beta$ in P. For every $\alpha \in P$ the neighborhoods $U_\alpha[x]$, $x \in X$, compose an entourage $U_\alpha = \{(x, y) \in X \times X : y \in U_\alpha[x]\}$ on X. The indexed family $\{U_\alpha\}_{\alpha\in P}$ is called a *P-base* for the topological space X. A P-base $\{U_\alpha\}_{\alpha\in P}$ is called *locally uniform* if for any point $x \in X$ and neighborhood $O_x \subset X$ of x there is $\alpha \in P$ such that the ball $U_\alpha U_\alpha^{-1} U_\alpha[x] = \{y \in X : (x, y) \in U_\alpha U_\alpha^{-1} U_\alpha\}$ is contained in O_x. It is clear that a topological space is first-countable if and only if it has an ω-base. By Moore Metrization Theorem, a topological space X is metrizable if and only if X is a T_0-space with a locally uniform ω-base. In the talk we shall discuss some properties of topological spaces possessing a (locally uniform) ω^ω-base. We show that topological spaces with an ω^ω-base share some common properties with first-countable spaces. In particular, many known upper bound on the cardinality of first-countable spaces remain true for countably tight spaces with an ω^ω-base. On the other hand, topological spaces with a locally uniform ω^ω-base have many properties, typical for generalized metric spaces.

References:

[1] T.Banakh, *Topological spaces with an ω^ω-base*,
preprint (http://arxiv.org/abs/1607.07978).

Bi-invariant metrics on diffeomorphism groups

Michael Brandenbursky
Ben-Gurion University of the Negev, Israel

Let Σ_g be a compact orientable surface of genus g and let $Ham(\Sigma_g)$ be the group of Hamiltonian diffeomorphisms of Σ_g. The most natural word metric on this group is the autonomous metric. It has many interesting properties, most important of which is the bi-invariance of this metric. In this talk I will show that for each g the group $Ham(\Sigma_g)$ is unbounded with respect to this metric. This is a joint work with Kedra and Shelukhin.

Polyhedral excuses

Jesús Castillo
University of Extremadura, Badajoz, Spain

A Banach space is said to be polyhedral if the unit ball of every finite dimensional subspace is the convex hull of a finite number of points. A Banach space is said to be *isomorphically polyhedral* if it admits a polyhedral renorming. Polyhedrality is a geometrical notion: c_0 is polyhedral while c is just isomorphically polyhedral.

Isomorphically polyhedral Banach spaces are elusive creatures, often making unexpected apparitions in remote areas. Vladimir Fonf has long since been the greatest polyhedral hunter, collector and scholar.

In this talk I though first about presenting a few open problems and examples in Banach space, such as the possibility of twisting $C(K)$-spaces or the existence of spaces of universal disposition, in which one way or another polyhedral creatures appear involved. So in the end I give up and openly consider the 3-space problem for isomorphic polyhedrality.

Gropes and their fundamental groups

Matija Cencelj
University of Ljubljana, Slovenia

Gropes are an important construction in geometric and algebraic topology, introduced by Stan'ko and Cannon. Open infinite gropes are nice 2-dimensional CW complexes and Eilenberg-MacLane spaces of their fundamental groups. We present some results on gropes and an application to comparing wildness of cetain crumpled cubes.

Topological transitivity and mixing of the composition operator on L^p spaces

Udayan B. Darji
University of Louisville, USA and Ashoka University, India

Let $X = (X, \Sigma, \mu)$ be a σ-finite measure space and $f : X \to X$ be an one-to-one bimeasurable transformation satisfying $\mu(f(B)) \geq c_1\mu(B)$ for some constant $c_1 > 0$ and every measurable set B. Then, $T_f : \varphi \mapsto \varphi \circ f$ is a bounded linear operator acting on $L^p(X, \Sigma, \mu)$, $1 \leq p < \infty$, called the *composition operator* induced by f. We provide necessary and sufficient conditions on f for T_f to be topologically transitive or topologically mixing. We also give two examples of one-to-one bimeasurable transformations whose composition operators are topologically transitive but not topologically mixing. Finally, we show that the composition operator induced by a bi-Lipschitz μ-contraction (or more generally, by a μ-dissipative transformation) defined on a finite measure space is always topologically mixing.

Extreme points in polyhedral Banach spaces

Carlo Alberto De Bernardi
University of Milano, Italy

A Banach space is called *polyhedral* if the unit ball of each of its finite-dimensional subspaces is a polytope. In 1966, Joram Lindenstrauss [2] proved that no infinite-dimensional dual (and in particular reflexive) Banach space is polyhedral. In view of this fact, the following problem, posed by Joram Lindenstrauss in [2], arises naturally.

Problem. *Does there exist a polyhedral infinite-dimensional Banach space whose unit ball is the closed convex hull of its extreme points?*

In [1] we solved in the affirmative the problem above. During the talk, we present the details of our construction.

References:

[1] C.A. De Bernardi, *Extreme points in polyhedral Banach spaces*, Israel J. Math., to appear.

[2] J. Lindenstrauss, *Notes on Klee's paper: "Polyhedral sections of convex bodies"*, Israel J. Math. **4** (1966), 235–242.

Self contracted curves on Riemannian manifolds

Robert Deville
University of Bordeaux, France

It is proved that a self contracted curve on a Riemannian manifold has finite length provided its range is relatively compact.

This is a joint work with A. Daniilidis, E. Durand-Cartagena and L. Rifford.

Making an F-space from an incomplete normed space

Tadeusz Dobrowolski
Pittsburg State University, USA

This is a joint work with W. Marciszewski.

For a normed linear space Y, we are interested in finding a stronger complete topology on Y such that Y becomes a complete metric linear space X (an F-space). Such an X, if it exists, must be unique. Moreover, if Y is separable, it must be Borel. In the realm of separable metric groups, Solecki showed that, for a separable Borel Abelian group Y, a stronger Polish group topology on Y *doesn't* exist iff there is a *topological* embedding $i : C^\omega \to \hat{Y}$ such that, for all $c, c' \in C^\omega$,

$$c - c' \in C^\omega_f \Leftrightarrow i(c) - i(c') \in Y,$$

that is, i sends the cosets of C^ω/C^ω_f into the cosets of $\hat{Y}/Y$ in a one-to-one manner. Above,

C^ω stands for the countable product of the Cantor group C, C^ω_f for eventually zero sequences, and $\hat{Y}$ for the completion of Y. We show that if Y is additionally a separable normed linear space, then the resulting Polish group X must be an F-space.

Several more observations on the subject will be discussed (this is a work in progress).

Non-separable Banach spaces studied via projectional skeletons and "rich" families of their separable subspaces

Marian Fabian
Institute of Mathematics, Academy of Science, Prague, Czech Republic

We present a structural statement characterizing Asplund space X via a certain "rich" family of separable "rectangles" $V \times Y \subset X \times X^*$. This enables to construct easily a "projectional skeleton" (a modern substitute of PRI) in X^*. Further we find some rich "rectangle" families in $X \times X^*$ where X is WCG, WLD, or even Plički space. Thus we get a "commutative" projectional skeleton in X. The class of Plička spaces is quite large. It contains $L_1(\mu)$, where μ is a σ-additive measure, duals of C^* algebras, order continuous lattices, $C(G)$, with G a compact abelian group, and preduals of semifinite von Neumann algebras. Finally, putting the results above together, we immediately yield that X is simultaneously Asplund and WCG, if and only if it admits a commutative projectional skeleton such that the adjoint projections form a projectional skeleton in X^*.

The lecture is based on three recent joint papers written together with Marek Cúth.

The Ascoli property for locally convex spaces

Saak Gabriyelyan
Ben-Gurion University of the Negev, Israel

Being motivated by the classic Ascoli theorem we introduced in [1] a new class of topological spaces, namely, the class of Ascoli spaces. In the talk we discuss the Ascoli property for various classes of locally convex spaces, Banach spaces and their unit balls endowed with the weak topology and function spaces.

References:

[1] T. Banakh, S. Gabriyelyan, *On the C_k-stable closure of the class of (separable) metrizable spaces*, Monatshefte Math. 180 (2016), 39-64.

Fibred finite asymptotic dimension and assembly maps

Boris Goldfarb
State University at Albany, USA

I will define a new weakening of Gromov's finite asymptotic dimension (FAD) for finitely generated groups which is called "fibred finite asymptotic dimension" and illustrate it with infinite-dimensional examples. There are now several different proofs of Novikov and related conjectures for groups with FAD. I will show that one of those methods can be extended to "fibred finite asymptotic dimension".

Coarse embeddings into $c_0(\Gamma)$

Petr Hájek
Institute of Mathematics, Academy of Science, Prague, Czech Republic

This is a joint work with Th. Schlumprecht.
We investigate the coarse embeddings into $c_0(\Gamma)$. We show e.g. that if X has density character at least $\aleph_\omega$ and X coarsely embeds into $c_0(\Gamma)$ then X cannot have nontrivial cotype.

On homomorphisms of topological groups

Stavros Iliadis
Lomonosov Moscow State University, Russia

We prove the following result:
Let **F** be an indexed collection of (open) topological homomorphisms of Raikov complete topological groups of weight less than or equal to a fixed infinite cardinal τ. Then, there exist a *continuously containing space* T_d of weight $\leq \tau$ for the indexed collection $\mathbf{S}_d$ of the domains of elements of **F** (see [1]) and a continuously containing space T_r of weight $\leq \tau$ for the indexed collection $\mathbf{S}_r$ of the ranges of the elements of **F** such that the mapping $F : \mathrm{X}_d \to \mathrm{Y}_r$, which is defined by relations $F|_X = f$, $X \in \mathbf{S}_d$, where f is the element of **F** whose domain is X, is correctly defined, (open), and continuous.

Also, we indicate some pairs $(\mathbb{S}_d, \mathbb{S}_r)$ of classes of topological groups, defined by topological properties of **spaces**, such that for each indexed collection **F** of topological homomorphisms $F : X \to Y$, where $X \in \mathbb{S}_d$ and $Y \in \mathbb{S}_r$, the corresponding continuously containing spaces T_d and T_r have also these properties.

References:

[1] S. Iliadis, *On embedings of topological groups*, Fundamental and Applied Mathematics, Vol 20, No.2, 2015, pp. 105-112 (Russian)

Separable quotient problem for spaces $C_p(K)$ over compact spaces K

Jerzy Kąkol
University of Poznań, Poland

The classic Rosenthal-Lacey theorem asserts that an infinite dimensional Banach space $C(K)$ of continuous real-valued maps on a compact Hausdorff space K has a Hausdorff quotient either isomorphic to ℓ_2 or c_0. Very recently we proved that the space $C_p(K)$ endowed with the pointwise topology has a separable quotient algebra iff K has a denumerable closed subset. Hence $C_p(\beta\mathbb{N})$ lacks separable quotient algebras. This motivates the following natural question:
$(*)$ *Does $C_p(K)$ admit an infinite dimensional separable Hausdorff quotient over any compact Hausdorff space K?*
Our main theorem reduces problem $(*)$ to the case when K is a compact *Efimov space* (i.e.

K neither contains infinite converging sequences nor a copy of $\beta\mathbb{N}$). Some applications are provided.

Separated subsets in the unit sphere of a Banach space

Tomasz Kania
University of Warwick, Great Britain

Let X be a Banach space. We study the circumstances under which there exists an uncountable set $\mathcal{A} \subset X$ of unit vectors such that $\|x - y\| > 1$ for distinct $x, y \in \mathcal{A}$. We prove that such a set exists if X is quasi-reflexive and non-separable; if X is additionally super-reflexive then one can have $\|x - y\| \geqslant 1 + \varepsilon$ for some $\varepsilon > 0$ that depends only on X. If K is a non-metrisable compact, Hausdorff space, then the unit sphere of $X = C(K)$ also contains such a subset; if moreover K is perfectly normal, then one can find such a set with cardinality equal to the density of X; this solves a problem left open by S. K. Mercourakis and G. Vassiliadis. These are joint results with Tomasz Kochanek (Warsaw).

Banach-Stone type theorem for C^1-function spaces over Riemannian manifolds

Kazuhiro Kawamura
University of Tsukuba, Japan

For a compact connected Riemannian manifold M, let $C^1(M)$ be the space of all real-valued C^1-functions on M with the C^1-topology. For a submanifold K of M, we define a norm $\|\cdot\|_{<M,K>}$ on $C^1(M)$ by $\|f\|_{<M,K>} = \max(\|f|K\|_\infty, \|df\|_\infty)$ for $f \in C^1(M)$, where df denotes the derivative of f and $\|df\|_\infty$ is the supremum of the operator norm $\|d_x f\|$ of the derivative $d_x f : T_x M \to \mathbb{R}$.

For two submanifolds K and L of M with $\dim K = \dim L > 0$, we prove that every surjective isometry $T : (C^1(M), \|\cdot\|_{<M,K>}) \to (C^1(M), \|\cdot\|_{<M,L>})$ such that $T(0) = 0$, $T(\mathrm{Const}) = \mathrm{Const}$ and $T(C^3(M)) = C^3(M)$ must be of the form: $Tf = \epsilon \cdot (f \circ \varphi), f \in C^1(M)$, where $\epsilon = \pm 1$ and $\varphi : M \to M$ is a Riemannian isometry with $\varphi(L) = K$.

Thus such isometries determine the embedding type of the submanifolds up to isometry. Some extensions and applications of the above will also be discussed.

Generalized Covering Spaces

James Keesling
University of Florida, USA

This is a joint work with Jerzy Dydak, Joanna Furno. We report on three results.

(1) Let G be an Abelian pro-finite group with underlying space homeomorphic to the Cantor. Then there is a separable metric space X with $\pi_1(X) \cong G$ as a group such that there is a surjective map of separable infinite-dimensional Hilbert space, $g : \ell_2 \to X$ which serves as a generalized universal covering map for X. Furthermore, the group of transformations $\mathcal{T}_g(\ell_2) \subset \mathcal{H}(\ell_2)$, defined by $h \in (T_g)(\ell_2)$ if and only if $g \circ h = g$ is isomorphic to G and the

natural topology on this set of homeomorphisms yields the given pro-finite topology on G.

We say that *all loops are small* in a pointed space (X, x_0) if the space has the property that for every loop $\gamma : [0,1] \to (X, x_0)$ and every open set $x_0 \in U \subset X$, there is a loop $\gamma' : [0,1] \to (U, x_0)$ which is homotopic to γ in (X, x_0). It is known that for such a space, there are no generalized covering spaces. This leads to

(2) Let G be an Abelian pro-finite group as in (1). Then there is a pointed separable metric space (X, x_0) having all loops small such that $\pi_1(X, x_0) \cong G$. Key to these first two results is the result of Bessaga and Pełczyński that the space of Borel measurable functions $\mathcal{M}([0,1], X)$ has the property that $\mathcal{M}([0,1], X) \cong \ell_2$ for all complete separable metric spaces X having at least two points.

(3) Let G be any group and let (X, x_0) be a pointed topological space such that $\pi_1(X, x_0) \cong G$. Then there is a pointed space (Y, y_0) and an embedding $e : (X, x_0) \subset (Y, y_0)$ such that (Y, y_0) has all loops small and such that $e_* : \pi_1(X, x_0) \cong \pi_1(Y, y_0)$. The construction of (Y, y_0) can be used to show additional results. For instance, if G has non-measurable cardinality, then the spaces (X, x_0) and (Y, y_0) can both be taken to be compact Hausdorff. It is also the case that $e : (X, x_0) \to (Y, y_0)$ is a weak homotopy equivalence. The space (Y, y_0) in (3) is not metrizable. There is no such (Y, y_0) metrizable with $\pi_1(Y, y_0) \cong \mathbb{Z}$.

Homological properties of decomposition spaces

Akira Koyama
Waseda University, Japan

Since 1960 the decomposition space X of the Euclidean 3-space $\mathbb{R}^3$ by the Case-Chamberlin continuum C has been investigated and Shrikhande showed that X is not simply connected and not locally simply connected. Bogopolski and Zastrow showed that the first dimensional singular homology group of X is not trivial. We are discussing (local) Čech homological properties of X.

A counterexample in Extension Theory

Michael Levin
Ben-Gurion University of the Negev, Israel

Let G be the p-adic circle. We construct a 3-dimensional compact metric space X such that the cohomological dimension of X with the coefficients in G is 1 and the standard Moore space $M(G, 1)$ for G is not an absolute extensor of X. Related results are discussed.

Where can real-valued Lipschitz functions on $\mathbb{R}^n$ be non-differentiable?

Olga Maleva
University of Birmingham, Great Britain

There are subsets N of $\mathbb{R}^n$ for which one can find a real-valued Lipschitz function f defined on the whole of $\mathbb{R}^n$ and not differentiable at every point of N. Of course, by the Rademacher theorem any such set N is Lebesgue null, however, due to a celebrated result of D. Preiss from 1990 not every Lebesgue null subset of $\mathbb{R}^n$ gives rise to such a Lipschitz function f.

In this talk, I discuss sufficient conditions on a set N for such f to exist. As a corollary of the main result we show that every purely unrectifiable set U possesses a Lipschitz function non-differentiable on U in the strongest possible sense.

This is a joint work with D. Preiss.

On group structures on covering spaces over groups

Vlasta Matijević
University of Split, Croatia

In the theory of covering maps over topological groups the following question naturally arises: If $f: X \to Y$ is a covering map from a connected space X onto a topological group Y, is it possible to define a group operation on X in such a way that X becomes a topological group and f a homomorphism of groups. Recently, the question was answered in the negative by constructing infinite-sheeted covering maps over solenoids which do not admit such group operation on total spaces. However, the answer is positive in two particular cases: if Y is a locally path connected group or if f is an overlay map over compact group Y. Note that overlay maps are a certain proper subclass of covering maps. We extend positive answers to covering maps over locally compactly connected groups Y and to overlay maps over locally compact groups Y generalizing previously obtained results. In both cases the group operation on X is unique and is defined by lifting of chains of open sets.

This is a joint work with Katsuya Eda.

References:

[1] K. Eda and V. Matijević, *Covering maps over solenoids which are not covering homomorphisms*, Fund. Math. 221 (2013), 69-82.

[2] K. Eda and V. Matijević, *Existence and uniqueness of group structures on covering spaces over groups*, Fund. Math. (to appear).

Infinite-dimensional uniform polyhedra and classifying spaces of 0-dimensional groups

Sergey A. Melikhov
Steklov Mathematical Institute, Moscow, Russia

Classifying spaces of groups are usually considered to be well-defined up to homotopy equivalence. But just like topological groups are naturally uniform spaces, their classifying spaces may well be thought of as uniform spaces, well-defined up to uniform homotopy equivalence (i.e. a homotopy equivalence given by uniformly continuous maps and homotopies).

But is this additional structure of any use? Not that I'm aware of, as long as discrete groups or Lie groups are concerned. However, for 0-dimensional groups it does make some difference. The classifying space $B\mathbb{Q}_p$ of the additive group of p-adic numbers is cohomologically 3-dimensional, but in the uniform setting, it becomes cohomologically infinite-dimensional. The classifying space $B\mathbb{Z}_p$ of the additive group of p-adic integers is at most 2-dimensional with respect to any cohomology theory, but in the uniform setting, it becomes infinite-dimensional with respect to complex K-theory. A characteristic application of this phenomenon is a p-adic Borsuk–Ulam theorem, which says that there exists no uniformly continuous equivariant map from $E\mathbb{Z}_p$ to any compact metric space with a free action of $\mathbb{Z}_p$. The key word here is "uniformly", for without it this would have been a much, much more interesting result. It must be mentioned that there has been no progress in this subject in recent years because I was working on unrelated projects. There are some new hopes, however, of obtaining the desired applications, rather than just their uniform analogues.

The "uniform setting" mentioned above is based on infinite-dimensional uniform polyhedra. These can be used, in particular, to approximate (in the sense of inverse limits) uniform classifying spaces — as well as any other separable metrizable complete uniform spaces. The main feature of such approximation is that infinite-dimensionality is retained in the individual approximants, including on an algebraic level. This property is absolutely lacking in usual approximations of metrizable topological spaces by polyhedra. The theory of infinite-dimensional uniform polyhedra (arXiv:1109.0346) settles Isbell's 1964 "Research Problem B_2" and is in turn based on reworking much of basic PL topology in strictly combinatorial terms (arXiv:1208.6309) and on putting in order the subjects of quotients and ANRs in the setting of metrizable uniform spaces (arXiv:1106.3249).

In the talk, I intend to give a review of infinite-dimensional uniform polyhedra, and then, armed with it, discuss constructions of uniform classifying spaces. The latter topic is still on its way from ad hoc methods to a satisfactory understanding (which may or may not improve by May).

Separable Lindenstrauss spaces whose duals lack the weak* fixed point property for nonexpansive mappings

Enrico Miglierina
Universita Cattolica del Sacro Cuove, Milano, Italy

In this talk we study the w^*-fixed point property for nonexpansive mappings. First we show that the dual space X^* lacks the w^*-fixed point property whenever X contains an isometric copy of the space c of the convergent sequences. Then, the main result of our paper provides some characterizations of weak-star topologies that fail the fixed point property for nonexpansive mappings in ℓ_1 space. This result allows us to obtain a characterization of all separable Lindenstrauss spaces X inducing the failure of w^*-fixed point property in X^*. The key tool of our result is a detailed study of the hyperplanes of the space c.

The talk is essentially based on a joint paper with Emanuele Casini and Łukasz Piasecki, appeared in Studia Mathematica 238 (1) (2017), 1-16.

Extension Theory in Large Scale Geometry

Atish J. Mitra
Montana Tech of the University of Montana, USA

Classical extension theory deals with extensions of maps between topological spaces. In this talk we will discuss progress made in the extension theory of functions in various large scale categories, and will compare the results and techniques with that of classical extension theory.

Spaces of compact diagonal operators as Calkin algebras of Banach spaces

Pavlos Motakis
Texas A&M University, USA

The Calkin algebra of a Banach space Z is the unital Banach algebra $\mathcal{C}al(Z)$ defined as the quotient $\mathcal{L}(Z)/\mathcal{K}(Z)$ of the algebra of all bounded linear operators on Z over the ideal of all compact ones. We investigate the question of what types of unital algebras can occur as Calkin algebras. Given a Banach space X with a Schauder basis we denote by $\mathcal{K}_{\text{diag}}(X)$ the space of all compact and diagonal operators on X. We prove that there exists a Banach space $\mathfrak{X}_X$ so that the Calkin algebra of $\mathfrak{X}_X$ is isomorphic, as a Banach algebra, to $\mathcal{K}_{\text{diag}}(X) \oplus \mathbb{R}I$. This yields Banach spaces with interesting Calkin algebras, e.g., James' quasi reflexive Banach space and even a hereditarily indecomposable Banach algebra constructed by S. A. Argyros, I. Deliyanni, and A. Tolias. The space $\mathfrak{X}_X$ is of the form $(\sum \oplus X_k)$ where each X_k is a version of the Argyros-Haydon space and the outside norm is a modified Argyros-Haydon sum incorporating the norm of the space X.

Balanced Presentations of the Trivial Group and Geometry of 4-Dimensional Manifolds

Alexander Nabutovsky
University of Toronto, Canada

We will describe triangulations of S^4 that are enormously far from each other in the metric defined as the minimal number of bistellar transformations required to transform one of them into the other. The number of constructed triangulations grows exponentially with the number of simplices; the required number of bistellar transformations grows faster than the tower of exponentials of any fixed height. The proof involves a construction of "many" balanced presentations of the trivial group so that these presentations are "very different" from each other (and from the trivial presentation). "Balanced" means that the number of generators is equal to the number of relations. We also demonstrate that there exist trivial 2-knots in R^4 that can be untied only if one allows an enormous increase of complexity during an isotopy.

Joint work with Boris Lishak.

On stability of Lipschitz functions on $c_0(\Gamma)$

Matěj Novotný
Institute of Mathematics, Academy of Science, Prague, Czech Republic

It is a famous result by Gowers that every real Lipschitz function on the sphere of c_0 stabilizes on some infinite-dimensional subspace. We prove for every uncountable Γ, there is a real symmetric 1-Lipschitz function defined on the sphere of $c_0(\Gamma)$ which doesn't stabilize on any of its subspaces with density $dens c_0(\Gamma)$. However, we prove that in the case of equivalent norms on $c_0(\Gamma)$, the stability on some subspace with density $dens c_0(\Gamma)$ is obtained.

Principle of local reflexivity which respects nests of subspaces, and the nest approximation properties

Eve Oja
University of Tartu, Estonia

We discuss new forms of the principle of local reflexivity (PLR): their strength is that the local reflexivity operators respect given nests of subspaces. These PLR results are then systematically applied to obtain duality and lifting theorems for the nest approximation properties in Banach spaces.

Nest approximation properties were launched by Figiel and Johnson in *J. Funct. Anal.* 271 (2016), 566–576. This concept involves finite-rank approximating operators which leave invariant all subspaces in a given nest of closed subspaces of a Banach space.

This is a joint work with Silja Veidenberg. The work was motivated by the question of Bill Johnson (at "Aleksander Pełczyński Memorial Conference", 2014): do there exist versions of the PLR which respect nests of subspaces?

Some recent development on proximinality in Banach spaces

Tanmoy Paul
Indian Institute of Technology Hyderabad, India

Aim of this talk is to discuss the behavior of the notion proximinality (existence of best approximation) for few cases; viz. the transitivity of this notion or its variant through the subspaces, duality between *the intersection properties of balls* and proximinality, stability of various strengthenings of proximinality in function spaces and generalization of this property to *Chebyshev centre* for a closed convex set. This talk is based on the following articles.

References:

[1] T. Paul, *Various notions of proximinality in spaces of Bochner integrable functions* Adv. Operator Theory, **2**, no 1, (2017) 59–77.

[2] S. Lalithambigai et al, *Chebyshev centres and some geometry of Banach spaces*, J. Math. Anal. Appl., **499**, no 1, (2017) 926–938.

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Free topological groups and functorial fibrewise constructions

Petar Pavešić
University of Ljubljana, Slovenia

In fibrewise homotopy theory one often needs fibrewise versions of various functorial constructions on pointed topological spaces (e.g. loop-spaces, Pontryagin-Thom construction, etc.).

We will present a general approach to this problem that is based on the construction of free topological groups.

Szlenk indices, variations, applications and problems

Matias Raja
University of Murcia , Spain

The Szlenk index is a powerful tool in isomorphic theory of Banach spaces. We review the relationships between the ordinary Szlenk index, the convex Szlenk index and the dentabilty index, including the recent results on optimal bounds. It is well known that the existence of asymptotically uniformly convex dual norms is related to the Szlenk index, however the use of alternative measures of noncompactness gives new alike results. We will show that a variation of the Szlenk index provides asymptotically uniformly smooth renormings without appealing to the duality. A few applications to c_0 are given since it is “the most” asymptotically uniformly smooth space. Finally, we will show some applications to the nonlinear classification of Banach spaces.

Recovering a Compact Hausdorff Space X from the Compatibility Ordering on $C(X)$

Martin Rmoutil
University of Warwick, Great Britain

Let X and Y be compact Hausdorff spaces. Let $f, g \in C(X)$ where $C(X)$ denotes the space of continuous functions on X. We say that g dominates f in the *compatibility ordering* if g coincides with f on the support of f. Our main result states that X and Y are homeomorphic if and only if there exists a compatibility isomorphism $T : C(X) \to C(Y)$. We derive several classical theorems of functional analysis as easy corollaries to our result:

If X and Y are compact Hausdorff spaces, we obtain that they are homeomorphic whenever there exists a bijection $T : C(X) \to C(Y)$ satisfying one of the following conditions:

- T is a ring isomorphism (Gelfand–Kolmogorov);
- T is multiplicative (Milgram);
- T the ordinary pointwise ordering (Kaplansky);
- T is linear and $Tf \cdot Tg = 0$ whenever $f \cdot g = 0$ (Jarosz).

We also study the question of continuity of compatibility isomorphisms: Depending on the properties of X we either prove or disprove that all compatibility isomorphisms are continuous (in different senses).

Coauthor: Tomasz Kania.

Width of the homotopy and the length of a shortest geodesic

Regina Rotman

University of Toronto, Canada

Recall that the width of a homotopy is defined as the maximal length of a trajectory of a point during this homotopy.

We will discuss various estimates of the width of "optimal" homotopies contracting arbitrary closed curves on a Riemannian manifold M in terms of the diameter of M, its volume, and various bounds for curvatures of M. Note that if M is closed, then the upper bounds for the width of such "optimal" homotopies can be used to obtain upper bounds for the length of the shortest periodic geodesic on M.

Results about optimal homotopies contracting closed curves can be non-trivial already in dimension 2. For example, we will discuss the following theorem (joint with E. Chambers, G. Chambers , A. De Mesmay, T. Ophelders):

Let D be the Riemannian 2-disk of diameter d. Suppose that the boundary curve ∂D can be contracted to a point via closed curves of length less than L. Then for any point q on the boundary of D, there exists a homotopy of ∂D to q over loops based at q of length less than $2d + L$.

Simplicial Inverse Sequences in Extension Theory

Leonard R. Rubin
University of Oklahoma, USA

In extension theory, in particular in dimension theory, it is frequently useful to represent a given compact metrizable space X as the limit of an inverse sequence of compact polyhedra. We are going to show that, for the purposes of extension theory, it is possible to replace such an X by a better metrizable compactum Z. This Z will come as the limit of an inverse sequence of triangulated polyhedra with bonding maps that are simplicial with respect to these fixed triangulations, and that factor in a certain way. As has been proved by S. Mardešić, not every metrizable compactum can be represented by such a “simplicial” inverse sequence. There will be a cell-like map $\pi : Z \to X$, and we shall show that if K is a CW-complex with $X\tau K$, then $Z\tau K$.

The simplicial inverse sequence determining Z is subject to “adjustments.” Any such adjustment determines an inverse sequence whose limit maps onto X, and the fibers of such a map have explicit and useful descriptions as the limits of certain subsequences. We plan to exploit this technique as part of a unified approach involving proofs of the resolution theorems of Edwards-Walsh (cell-like), Dranishnikov ($\mathbb{Z}/p$-acyclic), and Levin ($\mathbb{Q}$-acyclic), and perhaps to obtain a new and different one more general than the latter.
Coauthor: Vera Tonić.

Splittings and selections

Pavel V. Semenov
Higher School of Economics, Moscow, Russia

For an n multivalued mappings $F_k : X \to Y_k, k = 1, 2, ..., n$, for a singlevalued mapping

$L : \oplus Y_k \to Y$ and for a continuous selection $f : X \to Y$ of the composite mapping $L \circ (\oplus F_k)$ an $n-$tuple $(f_1, ..., f_n)$ is said to be a *splitting* of f if $f_k : Y_k \to Y$ is a selection of $F_k, k = 1, 2, ..., n,$ and $f(x) = L(f_1(x); ...; f_n(x)), x \in X$. A very special case give the following mappings: $F_k(\cdot) \equiv A_k$, where A_k are convex subsets of a Banach space Y, $L : Y^n \to Y$ linear continuous operator and $f = id \mid_X,\ X = L(A_1, ..., A_n)$. Under these assumptions a splitting of $f = id|_X$ gives a singlevalued solution $(y_1, ..., y_n)$ of the classical linear equation $L(y_1, ..., y_n) = y,\ y_1 \in A_1, ..., y_n \in A_n$ which continuously depends on the data y.

A series of results on splittings for the case $n = 2$ was considered in [1], [2]. Here we present the generalization for the case of an arbitrary $n \in \mathbb{N}$. Theorem.
Splittings always exist for a lower semicontinuous convex- and closed valued mappings $F_k : X \to \mathbb{R}, k = 1, 2, ..., n$ on a paracompact domain and for linear mapping $L : \mathbb{R}^n \to \mathbb{R}$

Remark, that the induction with the base $n = 2$ in general doesn't work even for the reduction of $n = 3$ to $n = 2$. So the proof is based on a good tomography of convex *rectangular* subsets, i.e. sets which coincide with the Cartesian product of their projections onto the coordinate lines. As it pointed out in [3] the restriction $dimY = 1$ is necessary for a positive answer on existence of splittings with an arbitrary paracompact domains.

By returning to the case $n = 2$ we have that for every linear $L : Y_1 \oplus Y_2 \to \mathbb{R}^3$ with surjective restriction $L|_{Y_1}$ and with non-zero restriction $L|_{Y_2}$ there are convex compacta $C_i \subset Y_i,\ i = 1, 2$ such that the linear equation

$$L(y_1, y_2) = y, \quad y_i \in C_i, \quad y \in C = L(C_1, C_2)$$

has no singlevalued solutions $y_1 = y_1(y),\ y_2 = y_2(y)$ which are continuous with respect to data $y \in C$.

The proof exploitees the Minkowski sum of the convex compacta $K = \overline{conv}\{(\cos t, \sin t, t) : 0 \le t \le 2\pi\}$, see [3] and the segment $S = \{(0, 0, t) : 0 \le t \le 2\pi\}$. It appears that for any splitting $c = k + s, \quad c \in K + S,\ k \in K,\ s \in S$ the second item is discontinuous with respect to c at the point $c_0 = (1, 0, 2\pi)$. For the first example of such effect, see [1].

References:
[1] M. V. Balashov and D. Repovš, J. Math. Anal. Appl., **355** (2009), 277–278
[2] D. Repovš, P.V. Semenov, J. Math. Anal. Appl., **334** (2007), 646 – 655
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On the convexity of Chebyshev sets

Evgeny Shchepin
Steklov Mathematical Institute, Moscow, Russia

A subset of a metric space is called a *Chebyshev set* if every point from its complement has a unique nearest point from the set. In finite-dimensional Banach spaces with smooth unique sphere, the classes of Chebyshev sets and closed convex sets are known to coincide. (T. S. Mozkin, L. N.H. Bunt). To deal with the non-smooth setting, which is more challenging, we shall need the concepts of convexity and smoothness in a given direction. The main result is as follows: a Chebyshev set in a finite-dimensional Banach space is convex in any direction tangent to the unit sphere of the space.

Here, under a direction tangent to the unit sphere one means a direction such that any line of the direction meeting the boundary of the ball but missing the interior is tangent (i.e. contains a tangent ray).

For the two-dimensional case, the opposite result holds: a connected closed subset of a 2-dimensional Banach space which is convex relative to all tangent directions of its unit sphere is a Chebyshev set iff its boundary and the unit sphere do not contain a rectilinear segments of the same direction.

References:

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Large and Discrete Sets in Countable Topological Groups

Ol'ga V. Sipacheva
Lomonosov Moscow State University, Russia

The most important result of the report is the following recent theorem proved jointly with Evgenii Reznichenko.

Throughout, all topological groups are assumed to be Hausdorff.

Theorem. (Reznichenko, Sipacheva)

(i) *Any countable nondiscrete topological group whose identity element has nonrapid filter of neighborhoods contains a discrete subset with precisely one limit point.*

(ii) *If there are no rapid filters, then any countable nondiscrete Boolean topological group contains two disjoint discrete subsets for each of which the zero of the group is a unique limit point.*

(A filter $\mathbb{F}$ on ω is rapid if every function $\omega \to \omega$ is majorized by the increasing enumeration of some element of $\mathbb{F}$. The nonexistence of rapid filters is consistent with ZFC.)

This theorem and its consequences concerning countable topological groups with extremal properties answer, partially or completely, several old questions. In particular, they imply that the nonexistence of a nondiscrete separable extremally disconnected group, as well as that of a nondiscrete countable topological group in which all discrete subspaces are closed, is consistent with ZFC.

A key role in the proof is played by fat subsets of a group. Fatness is a new notion of largeness organically related to that of syndeticity, which originated in Ramsey theory and topological dynamics in the context of the additive semigroup of positive integers, in which syndetic sets are precisely those with bounded gaps.

In the study of countable topological groups with extremal properties, of most interest are Boolean groups (any extremally disconnected group and any Abelian or countable irresolvable group contains an open Boolean subgroup), and every Boolean group is free, being a linear space over the field $\mathbb{Z}_2$.

Theorem.

(i) *Any countable Boolean topological group has a closed discrete basis.*

(ii) *Any countable closed linearly independent set in an extremally disconnected Boolean group has at most one limit point.*

(iii) *It is consistent with ZFC that any countable closed linearly independent set in an extremally disconnected Boolean group is discrete.*

Finally, we present a curious consequence of the equivalence between the existence of an extremally disconnected free (Boolean) topological group and that of selective ultrafilters, which concerns selectivity-type properties of filters.

Boundaries, polyhedrality and smoothness

Richard Smith
University College Dublin, Ireland

We give a survey of the progress made in the fields of polyhedrality and smoothness in Banach spaces over the last few years, in collaboration with Prof. Fonf and other authors, and present some open problems.

Retractional skeletons on trees

Jacopo Somaglia
University of Milano, Italy

A non-commutative Valdivia compact space is a compact space with retractional skeleton. A compact space is Valdivia if and only if it has a commutative retractional skeleton. The two classes do not agree: the ordinal space $[0, \omega_2]$, endowed with the interval topology, is an easy and well-known example of a non-commutative Valdivia compact space that is not Valdivia. In this talk, I will recall the definition and some properties of the Coarse wedge topology on trees. A characterization of non-commutative Valdivia trees will be presented. This characterization will be used to give a negative answer to the following question:

Problem. *Let X be a non-commutative Valdivia compact space that does not contain any copy of the ordinal space $[0, \omega_2]$. Is X necessarily Valdivia?*

Coincidence class for several maps

Stanisław Spież

IMPAN, Polish Academy of Sciences, Poland

Given two maps $f, g : M \to N$ between connected closed orientable manifolds of the same dimension, the Lefschetz coincidence number of f and g is defined as

$$\Lambda_{f,g} = \sum (-1)^k \mathrm{Tr}(D_M \circ g^* \circ D_N^{-1} \circ f_*),$$

where f_* and g^* are the induced homomorphisms of the homology groups and the cohomology groups, respectively, with rational coefficients, and D_M and D_N are the Poincaré duality isomorphisms. Lefschetz proved that if the coincidence number is nonzero, then f and g have a coincidence point.

For maps $f_1, \ldots, f_k : X \to N$ from a topological space X into a connected closed n-manifold (even non-orientable) N, we define a cohomological class

$$L(f_1, \ldots, f_k) \in H^{n(k-1)}(X; (f_1, \ldots, f_k)^*(R \times \Gamma_N^* \times \cdots \times \Gamma_N^*))$$

in such a way that $L(f_1, \ldots, f_k) \neq 0$ implies that the set of coincidences

$$\mathrm{Coin}(\mathrm{f}_1, \ldots, \mathrm{f}_\mathrm{k}) = \{\mathrm{x} \in \mathrm{X} \mid \mathrm{f}_1(\mathrm{x}) = \cdots = \mathrm{f}_\mathrm{k}(\mathrm{x})\}$$

is non-empty. (Here, R is a principal ideal domain and $\Gamma(N)$ is the orientation system of N.)

We prove some properties and give some applications of the defined Lefschetz type coincidence class.

This a joint research with Thais Monis.

The Lusternik-Schnirelmann category of general spaces

Tulsi Srinivasan
Ashoka University, India

The Lusternik-Schnirelmann category (LS-category) is a topological invariant that has historically been studied for absolute neighbourhood retracts. I will talk about how the theory of the LS-category can be extended to Peano continua like the Menger and Pontryagin spaces. I will then discuss current work applying these results to the boundaries of hyperbolic groups with an aim to finding an analogue to the Bestvina-Mess formula. I will also talk about how these technqiues can be applied to topological complexity.

Weakly compact subsets of $C[0,1]$ and reflexive Banach lattices

Pedro Tradacete
University of Madrid, Spain

We show that for every weakly compact subset K of $C[0,1]$ with finite Cantor-Bendixson rank, there is a reflexive Banach lattice E and an operator $T : E \to C[0,1]$ such that $K \subseteq T(B_E)$. On the other hand, we exhibit an example of a weakly compact set of $C[0,1]$ homeomorphic to $\omega^\omega + 1$ for which such T and E cannot exist. This answers a question of M. Talagrand in [Proc. Amer. Math. Soc. 96 (1986), no. 1, 95–102].

Joint work with J. Lopez-Abad.

Strongly Extreme Points and Approximation Properties

Stanimir Troyanski
University of Murcia, Spain

We establish, in terms of approximation properties, sufficient conditions for a strongly extreme point of a closed convex bounded subset of a Banach space to be a denting point. The results are from a joint work with T. Abrahamsen, P. Hajek and O. Nygaard.

Homogeneity of Erdős type space

Jan van Mill
University of Amsterdam, Holland

For subsets X of the real line we investigate homogeneity properties of the Erdős type space $E(X) = \{p \in \ell^2 : (\forall\, n)(p_n \in X)\}$. This space has much in common with the countable infinite product of copies of X, which is homogeneous by the result of Lawrence. Continuous families of coordinate permutations form an important ingredient in his proof. The Erdős space $E(X)$ is 1-dimensional in many cases which is an obstacle in homogeneity issues. On the other hand, $E(X)$ is invariant under coordinate permutations of ℓ^2 which suggests to investigate whether the Lawrence ideas are applicable. Our main result is the construction of a subset X of the real line such that every homeomorphism f of $E(X)$ is norm-preserving. That is, $\|f(p)\| = \|p\|$ for every $p \in E(X)$. Hence $E(X)$ need not be homogeneous. Since for every $\varepsilon > 0$, the sphere $\{p \in E(X) : \|p\| = \varepsilon\}$ is zero-dimensional, a natural question is whether spheres are always homogenous in Erdős type spaces. We prove that they are not.

Joint work with K. P. Hart.

Coarse dimension raising results

Žiga Virk
Institute of Science and Technology, Austria

The Hurewicz Dimension Raising Theorem states that a closed n-to-1 map between metric spaces may raise the covering dimension by at most $n-1$, i.e., $\dim Y \leq \dim X + n - 1$. We will survey the development of a coarse (asymptotic) version of this theorem by presenting the following results:

(1) *The first version of a Coarse Dimension Raising Theorem and a Finite-to-One Mapping Theorem* (joint with T. Miyata).

(2) *Improved bound via Gromov boundary and coarse permanence via coarsely n-to-1 maps* (joint with J. Dydak).

(3) *The optimal bound via Higson compactification and a generalization using coarse metric approximation* (joint with K. Austin).

Asymptotic dimensions related to some control functions

Michael Zarichnyi
University of Lviv, Ukraine and University of Rzeszów, Poland

The Gromov asymptotic dimension of a metric space X does not exceed n if, for any $D > 0$, there exists a uniformly bounded cover $\mathcal{U}$ of X which is the sum of $n+1$ D-disjoint families. If we impose restrictions onto the mesh of the cover $\mathcal{U}$ (as a function of D), we obtain versions of the asymptotic dimension, e.g., asymptotic Assouad - Nagata dimension [1] and asymptotic subpower dimension [2].

We consider some examples of spaces and groups with non-coinciding dimensions. We also consider the connections to the dimensions of the coronas (remainders of the compactifications) related to these asymptotic dimensions.

Some results are obtained in collaboration with Jacek Kucab.

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