

Set Theory, Model Theory and Applications

Abstracts of Plenary Talks

Finite homogeneous structures

Gregory Cherlin

Rutgers University, USA

A combinatorial or metric structure is *homogeneous* (in the sense of Fraïssé or Urysohn) if any isomorphism between finite parts is induced by an automorphism (with “isomorphism” read as “isometry” in the metric case).

There is an extensive theory of classification of homogeneous relational structures in finite languages developed by Lachlan with some technical assistance from Shelah, myself, and the guild of mathematicians who classified the finite simple groups. Lachlan discovered a fundamental connection between this problem and Shelah’s stability theory. He also proposed a generalization of his theory, which after further contributions from the group theorists led Hrushovski to introduce the methods of what is currently referred to as neostability into this context. This direction weakens the homogeneity condition, but again requires the language in question to be both finite and fixed (in a somewhat looser sense: this becomes a theorem rather than a hypothesis).

In a different direction, if we consider finite structures which are homogeneous in relational languages of bounded complexity (arity), we arrive at interesting combinatorial and group theoretic problems which are currently under investigation, and which I hope to discuss. In particular, I aim to touch on work of Dalla Volta, Gill, Hunt, Liebeck, Spiga, and Wiscons, much of it ongoing.

Global Structure Theorems for the space of measure preserving transformations

Matthew Foreman

University of California, Irvine, USA

Much of M. Rubin’s work was concerned with obtaining information about objects, or spaces of objects, by understanding their symmetries. In joint work with B. Weiss we prove a theorem in this spirit. We show that there is a very large class of ergodic transformations (a “cone” under the pre-ordering induced by factor maps) whose joining structure is identical to another class, the “circular systems”. The latter class is of interest because every member can be realized as a Lebesgue-measure preserving diffeomorphism of the torus \mathbb{T}^2 .

Using this theorem, we are able to conclude that the joining structure among diffeomorphisms includes that of a cone of diffeomorphisms. This solves several well-known problems such as the existence of ergodic Lebesgue measure preserving diffeomorphisms with an arbitrary

compact Choquet simplices of invariant measures and the existence of measure-distal diffeomorphisms of \mathbb{T}^2 of height greater than 2. (In fact we give examples of arbitrary countable ordinal height.)

Overlapping extenders and the Shelah Weak Hypothesis

Moti Gitik

Tel Aviv University, Israel

Extender based Prikry-Magidor forcing for overlapping extenders is introduced. As an application, a model in which the Shelah Weak Hypothesis for uncountable cofinality fails is constructed. Some other applications will be given.

Resolvability of topological spaces

István Juhász

Alfréd Renyi Institute of Mathematics, Hungarian Academy of Sciences, Budapest, Hungary

A topological space X is called λ -*resolvable*, where λ is a (finite or infinite) cardinal, if X contains λ many pairwise disjoint dense subsets. X is *maximally resolvable* if it is $\Delta(X)$ -resolvable, where

$$\Delta(X) = \min\{|G| : G \text{ open, } G \neq \emptyset\}.$$

The expectation is that “nice” spaces should be maximally resolvable, as verified e.g. by the well-known facts that both metric and linearly ordered spaces, as well as compact Hausdorff spaces, are maximally resolvable. There is, however, a countable regular (hence “nice”) space with no isolated points that is not even 2-resolvable.

In this talk we present resolvability results about spaces that are more general than the above. On the one hand, we consider the class of monotonically normal spaces that includes both metric and linearly ordered spaces, on the other we consider spaces with properties that are more general than compactness, for instance, Lindelöf, countably compact, and pseudocompact spaces.

Borel canonization and universally Baire sets

Menachem Magidor

Hebrew University, Jerusalem, Israel

The problem of Borel canonization was introduced by Kanovei, Sabok and Zapletal . In the original setting given an Analytic equivalence relation E and an idea I on the reals. The problem is to find a Borel set B which is not in the ideal such that E restricted to B is Borel. In this generality the answer is "NO", but if we put some "nicety" conditions on I and an the equivalence relation I one can get a positive answer, assuming some large cardinals.(These results are due to W. Chan and O. Drucker, independently).

In the talk we shall survey some possible generalizations of these results . For instance when we assume that the relation E is universally Baire.(Some of the results are joint results with W. Chan.)

The topology of some groups of homeomorphisms

Jan van Mill

University of Amsterdam, the Netherlands

We discuss some aspects of the topology of the homeomorphism groups $H(\mu^n)$ of the universal Menger continua μ^n . From the work of Mati Rubin it follows that for different n these groups are not algebraically isomorphic. Whether they are topologically homeomorphic is an open problem.

Subgroups of Richard Thompson's group F

Justin Moore

Cornell University, USA

Brin and Sapir have conjectured that every subgroup of Thompson's group F which does not contain a copy of F elementarily amenable. Until recently, however, there have been relatively few examples of complex subgroups of F which do not themselves contain a copy of F . In this talk I will describe a family of elementarily amenable subgroups of F which is strictly well-ordered by the embeddability relation in type ε_0 .

This is joint work with Collin Bleak, Matt Brin, Martin Kassabov, and Matt Zaremsky.

On existence of universal structures for classes of models

Saharon Shelah

Hebrew University, Jerusalem, Israel

The question is what can be the universality spectrum of a theory, i.e. the class of cardinals λ in which it has a universal model of cardinality λ (under elementary embeddings or under an embedding, the difference is negligible so far). Assuming, e.g. that the class is elementary, i.e. the class of models of a first order theory T , we know the answer for $\lambda = 2^{<\lambda}$ and that there are consistency results and some ZFC restrictions. Here we deal with some non existence ZFC results and consistency results.

Abstracts of Regular Talks

The divorce and possible reconciliation model theory and set theory

John T. Baldwin

University of Illinois at Chicago, USA

At least since Skolem's formulation of his paradox set theory and model theory have been intertwined. In contrast to Skolem, we investigate the methodological role of set theory in model theory. The very close connections in the 1960's were reflected by studies of Hanf numbers, two cardinal theorems etc. We argue that the advent of Shelah's classification theory resulted in a separation of set theory from a *first order* model theory focused on the study of particular theories and connections with traditional mathematics. Nevertheless, we isolate 4 types of entanglements between model theory and set theory. One is the just the use of infinitary combinatorics. The other three make more specific 3 connections with axiomatic set theory that we call 1) oracular, 2) meta-theoretic, and 3) entanglement. We will provide examples of the continued close connections with *infinitary* model theory and suggest directions of model theoretic research that could make the connections more profound.

Locally Moving Polymorphism Clones

Robert Barham

University of Exeter, UK

The reconstruction problem for first order models from their abstract polymorphism clones has attracted a lot of interest and progress in recent years. In this, I discuss one way in which Prof. Rubin's results about Locally Moving Groups can be used to study this problem, with particular application to the clones of the reducts of $(\mathbb{Q}, <)$.

Measures and randomisations of NSOP1 theories

Itai Ben Yaacov

University Claude Bernard, Lyon, France

The talk deals with applications of Keisler's randomisation construction to amalgamations of measures in simple (and NSOP1) theories. The randomisation associates to a theory T a theory T^R whose models are (more or less) spaces of random members of models of T . Equivalently, a type (over the empty set) of T^R is a probability measure on the space of types of T . If T is stable or NIP, then so is T^R , but if T is not NIP (e.g., simple unstable), then T^R has TP2 and cannot be even be simple.

Kaplan and Ramsey studied NSOP1 theories (which include the simple ones) and showed that they admit a notion of independence (Kim-independence) which is almost as good as forking independence in simple theories (and if T is simple, then the two agree). We prove that if T is NSOP1, then so is T^R , and characterise Kim-independence in T^R .

The independence theorem for Kim-forking in NSOP1 theories, applied in T^R , yields amalgamation results for measures in T .

Joint work with Artem Chernikov and Nick Ramsey.

Ramsey subgroups

Andreas Blass

University of Michigan, USA

30 years ago, I pointed out a close connection between the Boolean prime ideal theorem and a Ramsey property of group actions. Much more recently, I made two conjectures about this Ramsey property. Both conjectures were false: Dana Bartošová found a counterexample for one; Imre Leader, Mark Walters, and Christian Reiher found a counterexample for the other. I plan to describe the Ramsey property, my conjectures, the counterexamples, and improved conjectures that avoid the counterexamples.

Uniqueness triples from the diamond axiom

Ari Meir Brodsky

Ariel University, Israel

We work with a λ -*frame*, which is an abstract elementary class endowed with a collection of basic types and a non-forking relation satisfying certain natural properties with respect to models of cardinality λ .

We will show that assuming the diamond axiom $\diamond(\lambda^+)$, any basic type admits a non-forking extension that has a *uniqueness triple*. Prior results of Shelah in this direction required either some form of \diamond at two consecutive cardinals, or a constraint on the number of models of size λ . This is joint work with Adi Jarden.

This research was carried out with the assistance of the Center for Absorption in Science, Ministry of Aliyah and Integration, State of Israel.

Infinitary methods in finite combinatorics

James Cummings

Carnegie Mellon University, USA

Mostly a survey talk (with a few theorems) about two related notions of limit (one algebraic, one analytic) for convergent sequences of finite structures.

Ultrafilter on N and P-subfilters

Alan Dow

University of North Carolina at Charlotte, USA

A filter (base) F on N is a P-filter (base) if it is countably directed mod finite. A set a is a pseudo-intersection of a filter base F if a is infinite and $a \setminus b$ is finite for all $b \in F$. A filter F on N will be called nwd (nowhere dense) if it has no pseudo-intersection. A tower T will be a maximal collection of infinite subsets of N well-ordered by mod finite inclusion. Non-meager P-filters are nwd and towers are nwd P-filter bases. Interest in the question

(*) “Does every ultrafilter on N have a nwd P-subfilter?”

goes back to the early 1980’s because of the connection to two problems in topology. One remains open: Is there a product of sequentially compact regular spaces that is not countably compact? In 1980, the question (*) was shown to be independent: CH implies NO (Kunen, van Mill, Mills) and consistent behavior with respect to possible cofinalities of unbounded mod finite families in ω^ω yields models of YES (Balcar, Frankiewicz, Mills). NCF also implies a YES answer.

This inspired Nyikos to explore connections to natural questions about whether unbounded mod finite families in ω^ω are necessarily unbounded in ultrapower orderings. We answer one of his questions by establishing the connection to non-meager towers. Kunen, Medini and Zdomskyy have a 2015 paper “Seven characterizations of non-meager P-filters”. Non-meager P-filters were called fat P-filters in Shelah’s no P-points paper.

Back to (*): we obtain the consistency of a NO answer with not CH (and another question of Nyikos) with a novel(?) application of \square and also answer Nyikos’ question about the status in the Laver model.

A combinatorial notation for elementary topological properties

Misha Gavrilovich

St. Petersburg School of Economics and Management, Russia

We define a combinatorial notation which is able to express a number of elementary topological properties, for example compactness of Hausdorff spaces, being totally disconnected, connected, dense image, subspace, quotient, separation axioms. The notation is of category theoretic flavour and is based on finite topological spaces and the Quillen lifting property (orthogonality of morphisms) in the category of topological spaces; an expression denotes a class (property) of continuous maps. We end with some open questions: classify and find meaningful expressions; find the exact meaning of the expression for compactness for non-Hausdorff spaces; define a proof system our combinatorial expressions naturally live in.

The non-existence of universal minimal metric flows

Stefan Geschke

Hamburg University, Germany

We consider dynamical systems of the form (X, f) where X is a compact metric space and $f : X \rightarrow X$ is either a continuous map or a homeomorphism and provide a new proof that there is no universal metric dynamical system of this kind. The same is true for metric minimal dynamical systems and for metric abstract ω -limit sets, answering a question by Will Brian. Our proof uses a model-theoretic argument about group actions on Boolean algebras.

Separating cardinals in Cichon's diagram

Martin Goldstern

TU Wien, Austria

In a recent paper with Jakob Kellner and Saharon Shelah, we constructed a model where all cardinals in Cichon's diagram are different (as far as allowed by ZFC). I will explain a fragment of this proof, which will hopefully also be useful elsewhere: Strong witnesses.

The dp -rank of abelian and 1-based groups

Yatir Halevi

Hebrew University, Jerusalem, Israel

In this talk we will give a characterization of the dp -rank of 1-based groups. We will use this to describe a formula for the dp -rank of abelian groups. If time permits we will connect it to the vc -density of 1-based and abelian groups

A theory of pairs for non-valuational structures

Assaf Hasson

Ben-Gurion University of the Negev, Beer Sheva, Israel

Wencel showed that for a weakly o-minimal structure M the definable Dedekind completion of M can be endowed with an o-minimal structure M^* inducing on M the same definable sets as the one we started with. Wencel's construction is non-elementary in the sense that M^* depends on the structure M (and not on its theory). We associate to M^* a canonical language and then prove that $Th(M)$ determines $Th(M^*)$. We then investigate the theory of the pair $(M^*; M)$ in the spirit of the theory of dense pairs of o-minimal structures, and prove, among other results, that it is near model complete, and every definable open in M^* is already definable in M . This is joint work with E. Bar-Yehuda and Y. Peterzil.

Definable equivariant retractions onto skeleta in non-archimedean geometry

Martin Hils

Universität Münster, Germany

For a quasi-projective variety V over a non-archimedean valued field, Hrushovski and Loeser recently introduced a pro-definable space \widehat{V} , which is a model-theoretic analogue of the Berkovich analytification of V . They showed that \widehat{V} admits a pro-definable strong deformation retraction onto a skeleton, i.e., onto a space which is internal to the value group and thus piecewise linear.

If the underlying variety is an algebraic group G , the group G naturally acts on \widehat{G} by translation. In the talk, we will sketch the construction of an equivariant pro-definable strong deformation retraction of \widehat{G} onto a skeleton, in case G is a semiabelian variety. This is joint work with Ehud Hrushovski and Pierre Simon.

Iterated ultrapowers as countably compact groups

Michael Hrusak

Institute of Mathematics, Morelia, Mexico

We use the ultrapower construction to produce various examples of countably compact topological groups, answering thus old problems of Comfort and van Douwen. This is joint work with with Jan van Mill, Ariet Ramos and Saharon Shelah.

NIP henselian valued fields

Franziska Jahnke

Universität Münster, Germany

We investigate the question which henselian valued fields are NIP. In equicharacteristic 0, this is well understood due to the work of Delon: an henselian valued field of equicharacteristic 0 is NIP (as a valued field) if and only if its residue field is NIP (as a pure field). For perfect fields of equicharacteristic p , a characterization can be obtained by combining the work of Blair and Kaplan-Scanlon-Wagner. In this talk, I will present a characterization for henselian fields (K, v) to be NIP as long as the residue fields of all coarsenings of v have finite degree of imperfection. In particular, we will construct examples of NIP henselian fields with imperfect residue fields.

This is joint work with Sylvie Anscombe.

On some topological concepts related with tightness

Jerzy Kąkol

University of Poznan, Poland

It is known [Hrusak-Ramos Garcia] that *there exists a model in ZFC where every separable Fréchet-Urysohn group is metrizable*. This might motivate questions about good sufficient conditions under which a Fréchet-Urysohn group is metrizable. We provide some new and classic results about the (strong) Pytkeev property, the property which is essentially located between metrizability and countably tightness, including relations with metrizability and the Fréchet-Urysohn property for topological groups. Our approach is based on the concept (originally introduced by Cascales-Kąkol-Saxon) which is known under the name a \mathfrak{G} -base.

Recall that a family $\mathcal{U} := (U_\alpha)_{\alpha \in \mathbb{N}^{\mathbb{N}}}$ of subsets in a topological group G is called a \mathfrak{G} -base if \mathcal{U} is a base of neighbourhoods of the unit of G and $U_\beta \subseteq U_\alpha$ with $\alpha \leq \beta$ for all $\alpha, \beta \in \mathbb{N}^{\mathbb{N}}$.

An elementary proof of a theorem of Adler, Casanovas and Pillay

Itay Kaplan

Hebrew University, Jerusalem, Israel

I'll give an elementary proof of the following theorem: if p is a complete stable type over a set B which does not fork over a set A , then the restriction of p to A is also stable (this was first proved by Adler, Casanovas and Pillay using generically stable types). I will also discuss the NIP case and its connection to the stable case. Only the most basic knowledge of model theory will be assumed.

This is joint work with Pedro Andres Estevan Ibanez.

The Meta-Structure of Universes

Asaf Karagila

University of East Anglia, UK

We will discuss some expected and unexpected properties of the lattice of models of ZF and ZFC with a fixed class of ordinals. We will focus on the particularly peculiar example of *the Bristol model*: a model witnessing the unexpected depth of a single Cohen real over L .

Model theory of Galois actions

Piotr Kowalski

Wroclaw University, Poland

For a fixed finitely generated group G , we consider actions of G by field automorphisms. If the theory of such generic actions is first-order axiomatizable, then we say that the theory G -TCF exists. It is well-known that G -TCF exists if G is a free group (the theory ACFA), and it is also known that G -TCF exists for a finite G . On the other hand, it is also known that $(\mathbb{Z} \times \mathbb{Z})$ -TCF does not exist. Using Bass-Serre theory, we prove that if G is virtually free, then G -TCF exists generalizing the existing results about free groups and finite groups. We show that the new theories we obtain are not simple and not even NTP_2 .

This is joint work with Özlem Beyarslan.

Generic objects

Wieslaw Kubis

Institute of Mathematics of the Czech Academy of Sciences, Prague, Czech Republic

We shall present a rather general theory of objects resulting from certain infinite evolution processes. Namely, an evolution process is a sequence of objects (states) and transitions (morphisms) between them. There is a precise definition of limit of an evolution process. Certain property of the process implies that its limit is unique up to isomorphisms and typically has a rich automorphism group. We shall sketch the main technical results, followed by several new examples from different areas of mathematics. Our work extends both the classical and the new continuous Fraïssé theory.

Automorphism groups of lexicographically ordered chains, Hahn groups and fields

Salma Kuhlmann

University of Konstanz, Germany

Any automorphism of the exponent chain Γ combined with one of the base chain Δ induces an automorphism of the lexicographic chain Δ^Γ . This defines a homomorphism from the automorphism group of Δ^Γ onto the semidirect product of the automorphism groups of Γ and Δ . We study the image and kernel of this homomorphism. The kernel is the normal subgroup consisting on internal automorphisms. Ordered Hahn groups and fields (i.e. fields of generalized power series) are supported on lexicographic chains.

Inspired by Schilling's work [O. F. G. Schilling. *Automorphisms of fields of formal power series*. Bull. Amer. Math. Soc. 50.12 (1944), pp. 892-901] on the study of internal automorphisms of the field of Laurent series, we extend our investigation to ordered group automorphisms, respectively ordered field automorphisms of Hahn groups respectively Hahn fields.

Orbits of families of entire functions

Ashutosh Kumar

Hebrew University, Jerusalem, Israel

Erdős asked if there is a family \mathcal{F} of entire functions of size continuum such that for each complex number z , the set $\{f(z) : f \in \mathcal{F}\}$ has size less than continuum. We'll show that the answer is undecidable in ZFC plus the negation of CH.

This is joint work with Saharon Shelah.

Unbounded functions and infinite productivity of the Knaster property

Chris Lambie-Hanson

Bar-Ilan University, Israel

Motivated by questions about the circumstances under which the κ -Knaster property can be infinitely productive, we study principles asserting the existence of functions exhibiting certain strong unboundedness properties. We introduce the principle $U(\kappa, \lambda, \theta, \chi)$, which asserts the existence of a function $c : [\kappa]^2 \rightarrow \theta$ such that, for every family \mathcal{A} of κ -many pairwise disjoint elements of $[\kappa]^{<\chi}$ and every $i < \theta$, there is $\mathcal{B} \subseteq \mathcal{A}$ of size λ such that, for all $b, b' \in \mathcal{B}$ with $\sup(b) < \min(b')$, we have $\min(c[b \times b']) > i$. We derive instances of $U(\dots)$ from failures of stationary reflection, and we discuss applications to the infinite productivity of the κ -Knaster property and to the topological question of the tightness of the square of the sequential fan.

This is joint work with Assaf Rinot.

Products of homogeneous structures

Nadav Meir

Ben-Gurion University of the Negev, Beer Sheva, Israel

We will define the “lexicographic product” of two structures and show that if both structures admit quantifier elimination, then so does their product. As a corollary we get that nice (model theoretic) properties such as (ultra)homogeneity, homogenizability, stability, NIP and more are preserved under taking such products.

It is clear how to iterate the product finitely many times, but we will introduce a new infinite product construction which, while not preserving quantifier elimination, does preserve (ultra)homogeneity. As time allows, we will use this to give a negative answer to the last open question from a paper by A. Hasson, M. Kojman and A. Onshuus who asked “Is there a rigid elementarily indivisible* structure?”

* A structure M is said to be elementarily indivisible structure if for every coloring of its universe in two colors, there is a monochromatic elementary substructure N of M such that N is isomorphic to M .

The geometry of the pure n -ary ab initio Hrushovski construction

Omer Mermelstein

Ben-Gurion University of the Negev, Beer Sheva, Israel

Ab initio Hrushovski constructions are sparse hypergraphs constructed as a limit of a Fraïssé amalgamation class. Each such hypergraph naturally induces a combinatorial pregeometry (infinite finitary matroid) on its set of vertices.

In this talk we show that some of these combinatorial geometries are themselves limits of Fraïssé classes, and discuss some of their properties.

The Separable Quotient Problem for Topological Groups

Sidney A. Morris

La Trobe University and Federation University Australia (and regular visitor to Ben-Gurion University of the Negev)

One of the oldest problems in Banach Space theory is: Does every infinite-dimensional Banach space have a quotient space which is an infinite-dimensional separable Banach space? While it has been shown to be true for many important classes of Banach spaces, it remains an open question. The analogous problem for locally convex spaces asks: Does every infinite-dimensional locally convex space have a quotient space which is an infinite-dimensional space? Once again this is known to be true for many important classes. However, Jerzy Kakol, Steve Saxon and Aaron Todd, in 2014, proved that it is not true even for all barrelled spaces.

Over the last year or so, Arkady Leiderman, Mikhail Tkachenko and I have examined the analogous problem for topological groups. The first task was to decide what exactly is the "right" question or questions. I will report on our results. These results again cover many important classes.

(At this point I was tempted to say "some new and interesting results". But I recalled the dissertation examiner who said that the results in the thesis under examination were new and interesting, but unfortunately the interesting results were not new and the new results were not interesting.)

Definable amenability and fixed point theorems

Alf Onshuus

Universidad De Los Andes, Colombia

The strongest definition of an amenable topological group, is that the group admits a (left) translation-invariant probability measure on its Borel subsets. This notion was shown to be equivalent to having fixed points under particular actions from the group on certain topological spaces. Such "fixed point theorems" have been used in turn to generalize the concept of amenability outside locally compact groups.

In this talk, we will give several definitions of definable amenability for a definable group G , and relate them to fixed point theorems using the σ -topology generated by definable sets.

Automorphisms Groups and Reconstruction Problems

Gianluca Paolini

Hebrew University, Jerusalem, Israel

We give an overview of recent results on the theory of automorphism groups of countable structures, centered around the problem of reconstruction of model-theoretic properties of a structure from the properties of its automorphism group. On the positive side, we prove a general sufficient condition for strong small index property (SSIP) and a new reconstruction up to bi-definability result for structures with SSIP, in analogy with Rubin's well-known result for structures admitting a $\forall\exists$ -interpretation. On the negative side, we show that no topological or algebraic property of the group of automorphisms of a countable structure can detect any form of stability.

Reconstructing structures from their abstract clones

Michael Pinsker

Technische Universität, Wien and Charles University, Prague

To every countable first-order structure Δ , one can assign an algebra called its *polymorphism clone*, which has the same domain as Δ and whose functions are the homomorphisms from finite powers of Δ into itself. This algebra carries considerable information about Δ , particularly so when Δ is ω -categorical.

The polymorphism clone of Δ carries a natural topology, and certain properties of Δ can be recovered from the algebraic (equational) and topological structure of the clone, without knowledge of the actual algebra. We investigate when such properties can even be recovered from the purely algebraic information in the clone, disregarding its topology.

On isomorphism of κ -dense sets of reals and some related problems

Denis I. Saveliev

Institute for Information Transmission Problems, Moscow, Russia

Soon after J. Baumgartner's pioneer work showing consistency of order isomorphism of all \aleph_1 -dense sets of reals, other prominent set-theorists, including M. Rubin, established several further results in this topic. In my talk, I'll recall these results and say about some more recent advances and related problems.

An application of o -minimality in mathematical economics

Charles Steinhorn

Vassar College, USA

This talk deals with preference and utility theory in the context of o -minimal expansions \mathcal{R} of the ordered field of real numbers. We give a description of all preferences that can be defined in such a structure \mathcal{R} and when such preferences admit a utility function.

On Rubin's 'Theories of linear order'

Predrag Tanović

Mathematical Institute SANU and Faculty of Mathematics, Belgrade, Serbia

In his 1974 paper 'Theories of linear order' Rubin developed filigree techniques for analysing first order properties of coloured orders. He applied these techniques to determine the number of countable models of a complete $L_{\omega,\omega}$ theory T of infinite coloured (linear) orders and to study such T 's having at most countably many complete 1-types. He proved:

- (i) $I(\aleph_0, T)$ is either finite or 2^{\aleph_0} ;
- (ii) If the language is finite, then $I(\aleph_0, T) \in \{1, 2^{\aleph_0}\}$;
- (iii) If $|S_1(T)| \leq \aleph_0$, then $|S_n(T)| \leq \aleph_0$ for all n ;
- (iv) If $CB(S_1(T)) < \nu$, then $CB(S_2(T)) < \nu^4 \cdot 4 + 20$ (CB -Cantor-Bendixson rank);
- (v) If $S_1(T)$ is finite, then T is finitely axiomatizable.

The talk will contain analysis of Rubin's techniques in order to:

- (1) Describe definable sets in coloured orders;
- (2) Find conditions characterizing coloured orders among ordered structures;
- (3) Determine the number of countable models of a weakly o -minimal theory (or even dp -minimal, ordered).

Some recent results (joint with Slavko Moconja and partly with Dejan Ilić) will be presented: possible solutions to (1)–(2) and a partial solution of (3).

The Haar Measure Problem

Boaz Tsaban

Department of Mathematics, Bar-Ilan University, Ramat Gan, Israel

Every infinite compact group has a unique translation-invariant probability measure, its *Haar measure*. An old problem asks whether every compact group has a Haar-nonmeasurable subgroup. A series of earlier results reduce the problem to infinite metrizable profinite groups. Measure theoretically, these groups are just the Cantor space, equipped with the product probability measure but, surprisingly, the problem is still open.

Brian and Mislove observed that if the Continuum Hypothesis fails in some strong sense, then the answer is positive. We provide a positive answer assuming a weak, potentially provable, *consequence* of the Continuum Hypothesis.

Our hypothesis can be verified directly for some groups, including all abelian groups. Perhaps someone in the audience may help figuring out whether our weak hypothesis is, in fact, provable.

This is joint work with Adam J. Przeździecki and Piotr Szewczak.

Guessing models and the approachability ideal

Boban Veličković

Equipe de Logique, IMJ-PRG, Université Paris Diderot, Paris, France

Given a regular κ , recall that a κ -*guessing model* is a model of M size κ such that, for any ordinal $\alpha \in M$, any subset X of α that is κ -approximated in M is guessed by M . The principle $\text{ISP}(\kappa^+)$ was introduced by Viale and Weiss in 2011. It says that for any regular $\theta > \kappa$ there are stationary many κ -guessing models that are elementary submodels of H_θ . This principle implies that the two cardinal tree property $\text{ITP}(\kappa^+, \lambda)$, holds for any $\lambda \geq \kappa^+$, and much more. Starting with two supercompact cardinals, we describe a generic extension of the universe in which the principles $\text{ISP}(\omega_2)$ and $\text{ISP}(\omega_3)$ hold simultaneously, and the restriction of the approachability ideal $I[\omega_2]$ to the set of ordinals of cofinality ω_1 is the non stationary ideal on this set. This is an application of the theory of *virtual models* developed in order to iterate large classes of forcing notions.

This is joint work with my PhD student R. Mohammadpour.

Choiceless Ramsey Theory for Linear Orders

Thilo Weinert

University of Vienna, Austria

Ramsey-Type problems have been considered both in finite and infinite combinatorics. In infinite combinatorics most attention has been paid to partition relations between cardinals assuming the Axiom of Choice and almost all research dealt with ordinals (We think of cardinals as initial ordinals here). A notable exception is [1]. There the authors prove, among other things the following theorem.

Theorem. *Assume the Axiom of Choice. Then for all order-types t there is a colouring of the triples of t in black and white such that every quadruple contains a white triple and every copy of the integers contains a black one.*

This is similar to the folklore result (using AC) that for every order-type t there is a colouring of the pairs of t in black and white such that every infinite descending sequence contains a white pair and every infinite ascending sequence contains a white one. These two results put things into perspective. It is known that one can have very strong partition properties in models of ZF violating AC, consider for example Mathias's result that it is consistent with ZF that all sets of reals have the Ramsey property or Martin's discovery that AD implies that a colouring of the uncountable sets of countable ordinals with two colours admits an uncountable sets all of whose uncountable subsets get the same colour. This result failed to be published by him. We focus on linear orders of lexicographically ordered sequences of zeroes and ones of an ordinal length and prove positive and negative partition relations, an example of the latter is the following Theorem of ZF: For every initial ordinal κ and every ordinal α of cardinality at most κ there is a colouring of the quadruples of the linear order given by the lexicographically ordered sequences of zeroes and ones of length α in black and white such that every sextuple contains a black quadruple and each of the following order-types contains a white one:

$$\begin{aligned} &\kappa^* + \kappa \\ &2 + \kappa^* \\ &\kappa^2 \\ &\omega\omega^* \end{aligned}$$

We achieve a complete answer as to which partition relations for 2-colourings of points, pairs, triples or quadruples of the continuum are consistent with ZF. For larger linear orders however there are many questions left unanswered.

This is joint work with Philipp Lücke and Philipp Schlicht.

References.

[1] Paul Erdős, Eric Charles Miller and Richard Rado, *Partition relations for η_α -sets*, J. London Math. Soc. (2), 3:193–201, 1971.

Hereditarily Baire hyperspaces

Lyubomyr Zdomsky

Institute of Discrete Mathematics and Geometry, TU Wien, Austria

The talk is going to be devoted to the main ideas of the proof and applications of the following

Theorem. *Let X be a metrizable separable space. Then the space $\mathcal{K}(X)$ of all compact subspaces of X with the Vietoris topology is hereditarily Baire iff the remainder $\gamma X \setminus X$ of X in some (equivalently, any) compactification γX has the Menger covering property.*

The motivation behind this theorem comes among others from the study of families of compact sets and Tukey's ordering initiated by Christensen in 1974 and pursued by Fremlin [1] in 1991.

The talk will be based on a joint work in progress with Paul Gartside and Andrea Medini.

References.

[1] Fremlin, D.H., *Families of compact sets and Tukey's ordering*, Atti Sem. Mat. Fis. Univ. Modena **39**:1 (1991), 29–50.

On decidability of some classes of Stone algebras

Pavol Zlatoš

Comenius University, Bratislava, Slovakia

A typical (nontrivial) first order theory is undecidable. According to an early result of Tarski, the theory of Boolean algebras is one of the lucky exceptions. This was extended by Ershov to the decidability (of the theory) of relatively complemented distributive lattices as well as to Boolean algebras with a distinguished ultrafilter or prime ideal and to Post algebras. As shown by Rabin, even the second order theory of Boolean algebras with quantification over ideals is still decidable, implying the decidability of the first order theory of Boolean algebras with a sequence of distinguished ideals. On the other hand, several seemingly moderate generalizations of Boolean algebras are already undecidable. They include (bounded) distributive lattices (Grzegorzcyk), Boolean pairs, i.e., Boolean algebras with a distinguished subalgebra (Rubin), all varieties of Heyting algebras properly extending the variety of Boolean algebras (Burris), etc.

In our contribution we pursue examining the borderline between decidability and undecidability in the close vicinity of Boolean algebras. Iterating Katriňák's version of the Chen-Grätzer triple construction applied consecutively to n Boolean algebras we introduce the finitely axiomatizable classes \mathbf{SA}_n of n -th degree Stone algebras as follows. Given Boolean algebras B_1, B_2, \dots, B_n and their homomorphisms $h_i : B_i \rightarrow B_{i+1}$ ($1 \leq i < n$), referred to as the *structure maps*, we take the P -lattice

$$B_1 \times_{h_1} B_2 \times_{h_2} \dots \times_{h_{n-1}} B_n = \{(b_1, b_2, \dots, b_n) \in B_1 \times B_2 \times \dots \times B_n \mid \\ h_1(b_1) \geq b_2, h_2(b_2) \geq b_3, \dots, h_{n-1}(b_{n-1}) \geq b_n\},$$

regarded as a $(0, 1)$ -sublattice of the direct product $B_1 \times B_2 \times \dots \times B_n$, with the pseudocomplement operation

$$(b_1, b_2, \dots, b_n)^* = (b_1^*, h_1(b_1^*), \dots, (h_{n-1} \circ \dots \circ h_1)(b_1^*)).$$

Next we introduce two finitely axiomatizable subclasses \mathbf{SA}_n^i and \mathbf{SA}_n^s of the class \mathbf{SA}_n , with all the structure maps h_i in their P-lattice representation injective or surjective, respectively. Then the class of all Post algebras of degree n is definitionally equivalent to the intersection $\mathbf{SA}_n^i \cap \mathbf{SA}_n^s$. Taking the liberty of confusing first order languages, the class \mathbf{SAD}_n of all n -th degree Stone algebras which are dually pseudocomplemented and form a dual Stone algebra under the operation of dual pseudocomplement satisfies the inclusions $\mathbf{PA}_n \subseteq \mathbf{SAD}_n \subseteq \mathbf{SA}_n^s$.

Building on Rubin's undecidability result for the class of Boolean pairs we show that already the class \mathbf{SA}_2^i of all Stone algebras with Boolean dense elements set and injective structure map h_1 is hereditarily undecidable, hence all the classes \mathbf{SA}_n^i are undecidable for $n \geq 2$, too. The same is true for the classes \mathbf{SA}_n and the class of all *Gödel algebras*, i.e., Heyting algebras satisfying $(x \rightarrow y) \vee (y \rightarrow x) = 1$.

On the other hand, using Rabin's method of interpretation (semantic embedding) and his above mentioned result, we show that all the classes \mathbf{SA}_n^s are decidable. As a consequence we obtain the decidability of the classes \mathbf{SAD}_n , as well as another proof of Ershov's decidability result for the classes \mathbf{PA}_n . Finally, from a result of K. and P. Idziak, characterizing varieties of Heyting algebras with decidable first order theory of their finite members, it follows that the classes of all finite algebras in \mathbf{SA}_n are decidable for each n .

This is joint work with Martin Adamčík.