סטטיסטיקה מתמטית $\boxed{\text{תרגיל 1}}$

התפלגויות, מדגמים בגודל n.

1). נלקח מדגם $(X_1,\dots,X_n)$ מהתפלגות ברנולי. $X_i \sim B(1,p)$, $i=1,2,\dots,n$

נסמן: $S_n = X_1+\dots+X_n$, $\bar{X}_n = \frac{1}{n} S_n$

a). מצא את התפלגותם של S_n ו- $\bar{X}_n$.

b). חשב את מקדם המתאם $\rho(S_m, X_n)$ לכל m ו- n.

c). מצא את פונקצ"ת ההסתברות המותנית $f_{X_k|S_n}$ לכל k ו- n.

d). מצא את פונקצ"ת ההסתברות המשותפת $f_{X,Y}$ וגם את $f_{X,Y,Z}$ כאשר

$$X = X_1 + X_2 , \quad Y = X_1 + X_3 , \quad Z = X_2 + X_3$$

2). מצא את ההתפלגות של $S_n = X_1 + \dots + X_n$ כאשר $(X_1,\dots,X_n)$ מדגם מאחת מההתפלגויות הבאות:

a). $B(m,p)$, d). $Exp(\lambda)$

b). $G(p)$, e). $N(\mu,\sigma^2)$

c). $P(\lambda)$, f). $U(0,1)$, ~~$(n=2,3)$~~

h) $NB(m,p)$, g). $\Gamma(\alpha,\lambda)$

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3) מה ההתפלגות של $Y+X$ כאשר X ו Y ב"ת:

נתון: a) $X, Y \sim U_d(N)$.

b) $X \sim P(\lambda)$, $Y \sim P(\mu)$.

c) $X \sim U(-1,1)$, $Y \sim U(0,3)$.

d) $X \sim \Gamma(\alpha,\lambda)$, $Y \sim \Gamma(\beta,\lambda)$

e) $X \sim N(\mu_1,\sigma_1^2)$, $Y \sim N(\mu_2,\sigma_2^2)$.

תזכ

$$f_{X+Y}(t)=\int_{-\infty}^{+\infty} f_X(u)\cdot f_Y(t-u)\,du$$

4) מצא את ההתפלגות של X^2 כאשר

a) $X \sim U(-1,2)$, b) $X \sim N(0,1)$.

c) $X \sim Exp(\lambda)$. d) $X \sim \Gamma(\alpha,\lambda)$.

חשב את EX^2 בכל המקרים.

5) נלקח מדגם $(X_1,\dots,X_n)$ מההתפלגות $U(a,b)$

a) מצא את ההתפלגות של $\frac{1}{2}(X_1+X_2)$

b) מה ההתפלגויות של $\tilde{X}$ ו $\underset{\sim}{X}$ כאשר

$\underset{\sim}{X} = \min(X_1,\dots,X_n)$, $\tilde{X} = \max(X_1,\dots,X_n)$

c) חשב את $E\tilde{X}$ ו $E\underset{\sim}{X}$

Table 1 DISCRETE DISTRIBUTIONS

Name of parametric family of distributions	Discrete density functions $f(\cdot)$	Parameter space	Mean $\mu = \mathscr{E}[X]$	Variance $\sigma^2 = \mathscr{E}[(X-\mu)^2]$	Moments $\mu_r' = \mathscr{E}[X^r]$ or $\mu_r = \mathscr{E}[(X-\mu)^r]$ and/or cumulants κ_r	Moment generating function $\mathscr{E}[e^{tX}]$
Discrete uniform $U_d(N)$	$f(x) = \frac{1}{N} I_{\{1,\ldots,N\}}(x)$	$N = 1, 2, \ldots$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$	$\mu_3' = \frac{N(N+1)^2}{4}$ $\mu_4' = \frac{(N+1)(2N+1)(3N^2+3N-1)}{30}$	$\sum_{j=1}^{N} \frac{1}{N} e^{jt}$
Bernoulli $B(1, p)$	$f(x) = p^x q^{1-x} I_{\{0,1\}}(x)$	$0 \le p \le 1$ $(q = 1-p)$	p	pq	$\mu_r' = p$ for all r	$q + pe^t$
Binomial $B(n, p)$	$f(x) = \binom{n}{x} p^x q^{n-x} I_{\{0,1,\ldots,n\}}(x)$	$0 \le p \le 1$ $n = 1, 2, 3, \ldots$ $(q = 1-p)$	np	npq	$\mu_3 = npq(q-p)$ $\mu_4 = 3n^2p^2q^2 + npq(1-6pq)$	$(q + pe^t)^n$
Hypergeometric $H(M, K, n)$	$f(x) = \frac{\binom{K}{x}\binom{M-K}{n-x}}{\binom{M}{n}} I_{\{0,1,\ldots,n\}}(x)$	$M = 1, 2, \ldots$ $K = 0, 1, \ldots, M$ $n = 1, 2, \ldots, M$	$n\frac{K}{M}$	$n\frac{K}{M}\frac{M-K}{M}\frac{M-n}{M-1}$	$\mathscr{E}[X(X-1)\cdots(X-r+1)] = r!\frac{\binom{K}{r}\binom{n}{r}}{\binom{M}{r}}$	not useful
Poisson $P(\lambda)$	$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} I_{\{0,1,\ldots\}}(x)$	$\lambda > 0$	λ	λ	$\kappa_r = \lambda$ for $r = 1, 2, \ldots$ $\mu_3 = \lambda$ $\mu_4 = \lambda + 3\lambda^2$	$\exp[\lambda(e^t - 1)]$
Geometric $G(p)$	$f(x) = pq^{x-1} I_{\{1,2,\ldots\}}(x)$	$0 < p \le 1$ $(q = 1-p)$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\mu_3 = \frac{q+q^2}{p^3}$ $\mu_4 = \frac{q+7q^2+q^3}{p^4}$	$\frac{pe^t}{1-qe^t}$
Negative binomial $NB(m, p)$	$f(x) = \binom{x-1}{m-1} p^m q^{x-m} I_{\{m, m+1, \ldots\}}(x)$	$0 < p \le 1$	$\frac{m}{p}$	$\frac{rq}{p^2}$	$\mu_3 = \frac{r(q+q^2)}{p^3}$ $\mu_4 = \frac{r[q+(3r+4)q^2+q^3]}{p^4}$	

$$VS^2 = \frac{1}{n}\left(\mu_4 - \frac{n-3}{n-1}\sigma^4\right)$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2 = \frac{1}{n-1}\sum X_i^2 - \frac{n}{n-1}\bar{X}^2$$

$$B(a, b) = \frac{\Gamma(a)\cdot\Gamma(b)}{\Gamma(a+b)}$$

$$\Gamma(n) = (n-1)!$$

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Table 2 CONTINUOUS DISTRIBUTIONS

Name of parametric family of distributions	Cumulative distribution function $F(\cdot)$ or probability density function $f(\cdot)$	Parameter space	Mean $\mu = \mathscr{E}[X]$	Variance $\sigma^2 = \mathscr{E}[(X-\mu)^2]$	Moments $\mu_r' = \mathscr{E}[X^r]$ or $\mu_r = \mathscr{E}[(X-\mu)^r]$ and/or cumulants κ_r	Moment generating function $\mathscr{E}[e^{tX}]$
Uniform or rectangular $U(a, b)$	$f(x) = \frac{1}{b-a} I_{[a,b]}(x)$	$-\infty < a < b < \infty$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\mu_r = 0$ for r odd $\mu_r = \frac{(b-a)^r}{2^r(r+1)}$ for r even	$\frac{e^{bt}-e^{at}}{(b-a)t}$
Normal $N(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp[-(x-\mu)^2/2\sigma^2]$	$-\infty < \mu < \infty$ $\sigma > 0$	μ	σ^2	$\mu_r = 0$, r odd; $\mu_r = \frac{r!}{(r/2)!}\frac{\sigma^r}{2^{r/2}}$, r even; $\kappa_r = 0$, $r > 2$	$\exp[\mu t + \frac{1}{2}\sigma^2 t^2]$
Exponential $Exp(\lambda)$	$f(x) = \lambda e^{-\lambda x} I_{(0,\infty)}(x)$	$\lambda > 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\mu_r' = \frac{\Gamma(r+1)}{\lambda^r}$	$\frac{\lambda}{\lambda - t}$ for $t < \lambda$
Gamma $\Gamma(r, \lambda)$	$f(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x} I_{(0,\infty)}(x)$	$\lambda > 0$ $r > 0$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	$\mu_j' = \frac{\Gamma(r+j)}{\lambda^j \Gamma(r)} = \frac{\alpha(\alpha+1)\ldots(\alpha+r-1)}{\lambda^r}$	$\left(\frac{\lambda}{\lambda - t}\right)^r$ for $t < \lambda$
Beta $Beta(a, b)$	$f(x) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} I_{(0,1)}(x)$	$a > 0$ $b > 0$	$\frac{a}{a+b}$	$\frac{ab}{(a+b+1)(a+b)^2}$	$\mu_r' = \frac{B(r+a, b)}{B(a,b)}$	not useful
Cauchy	$f(x) = \frac{1}{\pi\beta\{1 + [(x-\alpha)/\beta]^2\}}$	$-\infty < \alpha < \infty$ $\beta > 0$	Does not exist	Does not exist	Do not exist	Characteristic function is $e^{i\alpha t - \beta\lvert t\rvert}$
Lognormal	$f(x) = \frac{1}{x\sqrt{2\pi}\sigma} \exp[-(\log_e x - \mu)^2/2\sigma^2] I_{(0,\infty)}(x)$	$-\infty < \mu < \infty$ $\sigma > 0$	$\exp[\mu + \frac{1}{2}\sigma^2]$	$\exp[2\mu + 2\sigma^2] - \exp[2\mu + 2\sigma^2]$	$\mu_r' = \exp[r\mu + \frac{1}{2}r^2\sigma^2]$	not useful
Double exponential	$f(x) = \frac{1}{2\beta}\exp\left(-\frac{\lvert x-\alpha\rvert}{\beta}\right)$	$-\infty < \alpha < \infty$ $\beta > 0$	α	$2\beta^2$	$\mu_r = 0$ for r odd; $\mu_r = r!\,\beta^r$ for r even	$\frac{e^{\alpha t}}{1-(\beta t)^2}$

(continued)

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx$$

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Table 2 CONTINUOUS DISTRIBUTIONS (*continued*)

Name of parametric family of distributions	Cumulative distribution function $F(\cdot)$ or probability density function $f(\cdot)$	Parameter space	Mean $\mu = \mathscr{E}[X]$	Variance $\sigma^2 = \mathscr{E}[(X-\mu)^2]$	Moments $\mu_r' = \mathscr{E}[X^r]$ or $\mu_r = \mathscr{E}[(X-\mu)^r]$ and/or cumulants κ_r	Moment generating function $\mathscr{E}[e^{tX}]$
Weibull W(a, b)	$f(x) = abx^{b-1}\exp[-ax^b]I_{(0,\infty)}(x)$	$a > 0$ $b > 0$	$a^{-1/b}\Gamma(1+b^{-1})$	$a^{-2/b}[\Gamma(1+2b^{-1}) - \Gamma^2(1+b^{-1})]$	$\mu_r' = a^{-r/b}\Gamma\left(1+\frac{r}{b}\right)$	$\mathscr{E}[X^t] = a^{-t/b}\Gamma\left(1+\frac{t}{b}\right)$
Logistic	$F(x) = [1+e^{-(x-\alpha)/\beta}]^{-1}$	$-\infty < \alpha < \infty$ $\beta > 0$	α	$\frac{\beta^2\pi^2}{3}$		$e^{\alpha t}\pi\beta t\csc(\pi\beta t)$
Pareto Par(θ, x_0)	$f(x) = \frac{\theta x_0^\theta}{x^{\theta+1}} I_{(x_0,\infty)}(x)$	$x_0 > 0$ $\theta > 0$	$\frac{\theta x_0}{\theta-1}$ for $\theta > 1$	$\frac{\theta x_0^2}{(\theta-1)^2(\theta-2)}$ for $\theta > 2$	$\mu_r' = \frac{\theta x_0^r}{\theta-r}$ for $\theta > r$	does not exist
Gumbel or extreme value	$F(x) = \exp(-e^{-(x-\alpha)/\beta})$	$-\infty < \alpha < \infty$ $\beta > 0$	$\alpha + \beta\gamma$, $\gamma \approx .577216$	$\frac{\pi^2\beta^2}{6}$	$\kappa_r = (-\beta)^r\psi^{(r-1)}(1)$ for $r \ge 2$, where $\psi(\cdot)$ is digamma function	$e^{\alpha t}\Gamma(1-\beta t)$ for $t < 1/\beta$
t distribution	$f(x) = \frac{\Gamma[(k+1)/2]}{\Gamma(k/2)}\frac{1}{\sqrt{k\pi}}\frac{1}{(1+x^2/k)^{(k+1)/2}}$	$k > 0$	$\mu = 0$ for $k > 1$	$\frac{k}{k-2}$ for $k > 2$	$\mu_r = 0$ for $k > r$ and r odd; $\mu_r = \frac{k^{r/2}B((r+1)/2, (k-r)/2)}{B(\frac{1}{2}, k/2)}$ for $k > r$ and r even	does not exist
F distribution	$f(x) = \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)}\left(\frac{m}{n}\right)^{m/2} \times \frac{x^{(m-2)/2}}{[1+(m/n)x]^{(m+n)/2}} I_{(0,\infty)}(x)$	$m, n = 1, 2, \ldots$	$\frac{n}{n-2}$ for $n > 2$	$\frac{2n^2(m+n-2)}{m(n-2)^2(n-4)}$ for $n > 4$	$\mu_r' = \left(\frac{n}{m}\right)^r\frac{\Gamma(m/2+r)\Gamma(n/2-r)}{\Gamma(m/2)\Gamma(n/2)}$ for $r < \frac{n}{2}$	does not exist
Chi-square distribution	$f(x) = \frac{1}{\Gamma(k/2)}\left(\frac{1}{2}\right)^{k/2} x^{k/2-1}e^{-(1/2)x}I_{(0,\infty)}(x)$	$k = 1, 2, \ldots$	k	$2k$	$\mu_j' = \frac{2^j\Gamma(k/2+j)}{\Gamma(k/2)}$	$\left(\frac{1}{1-2t}\right)^{k/2}$ for $t < 1/2$

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פתרון

(1 (a $S_n \sim B(n,p)$, $f_{S_n}(k)=\binom{n}{k}p^{k}q^{n-k}$
$k=0,1,2,\dots,n$

$$f_{\bar X_n}(x)=\begin{cases}\binom{n}{k}p^{k}q^{n-k}, & x=\frac{k}{n},\ k=0,1,2,\dots,n\\ 0, & x\neq \frac{k}{n}\end{cases}$$

(b $S_m\sim B(m,p)$, $S_n\sim B(n,p)$

נניח $n>m$; $S_n=X_1+\dots+X_n$

$X_i\sim B(1,p)$ בת"ל
$T_{m,n}=X_{m+1}+\dots+X_n$
$S_m=X_1+\dots+X_m$
S_m ו $T_{m,n}$ בת"ל

$$\begin{aligned}\operatorname{cov}(S_m,S_n)&=\\ &=\operatorname{cov}(S_m,S_m+T_{m,n})=\\ &=\operatorname{cov}(S_m,S_m)+\operatorname{cov}(S_m,T_{m,n})=\\ &=V(S_m)+0=mpq\end{aligned}$$

$$\rho(S_m,S_n)=\frac{mpq}{\sqrt{VS_m}\cdot\sqrt{VS_n}}=\frac{mpq}{\sqrt{mpq}\cdot\sqrt{npq}}=\sqrt{\frac{m}{n}}$$

$$\rho(S_m,S_n)=\begin{cases}\sqrt{\frac{m}{n}}, & m<n\\ 1, & m=n\\ \sqrt{\frac{n}{m}}, & m>n\end{cases}$$

(c $f_{X_k|S_n}(i|m)=P\{X_k=i\mid S_n=m\}$
$i=0,1$, $m=0,1,\dots,n$

אם $k>n$ X_k ו S_n בת"ל.

אם $1\le k\le n$

$$P\{X_1=1\mid S_n=m\}=\frac{P\{X_1=1,\ X_1+\dots+X_n=m\}}{P\{X_1+\dots+X_n=m\}}=$$

$$= \frac{P\{X_1=1\}\cdot P\{X_2+\dots+X_n=m-1\}}{P\{S_n=m\}} =$$

$$= \frac{p\cdot\binom{n-1}{m-1}p^{m-1}q^{(n-1)-(m-1)}}{\binom{n}{m}p^m q^{n-m}} = \frac{\binom{n-1}{m-1}}{\binom{n}{m}} = \frac{m}{n}$$

$\underline{1\le k\le n}$

$$f_{X_k|S_n}(i\,|\,S_n=m) = \begin{cases} \frac{m}{n}, & i=1\\ 1-\frac{m}{n}, & i=0\end{cases}$$

$$f_{X_k}(i) = f_{X_k|S_n}(i\,|\,m)$$

$\underline{k>n}$ ρ/c

$X_k \sim B(1,p)$

$$f_{X,Y}(x,y) = P\{X=x,\ Y=y\}$$

(d

$X,Y\sim B(2,p)$

$\underline{x,y=0,1,2}$

$\boxed{x_1x_2x_3}$

000	110
100	101
010	011
001	111

$x=x_1+x_2$

$y=x_1+x_3$

X \ Y	0	1	2	
0	q^3	q^2p	0	q^2
1	q^2p	$pq = q^2p+p^2q$	qp^2	$2pq$
2	0	qp^2	p^3	p^2
	q^2	$2pq$	p^2	1

$z=x_2+x_3$

$x,y,z=0,1,2$

$x_1x_2x_3$	000	001	010	100	011	101	110	111
~~x y z~~	000	011	101	110	112	121	211	222
$f(x,y,z)$	$\cancel{q^3}$	$\cancel{q^2p}$	$\cancel{q^2p}$	q^2p	p^2q	p^2q	p^2q	p^3

$$f(x,y,z) = P\{X=x,\ Y=y,\ Z=z\}$$

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2) a) $S_n \sim B(mn, p)$

b) $S_n \sim NB(n, p)$

c) $S_n \sim P(n\lambda)$

d) $S_n \sim \Gamma(n, \lambda)$

e) $S_n \sim N(n\mu, n\sigma^2)$

g) $S_n \sim \Gamma(n\alpha, \lambda)$

h) $S_n \sim NB(mn, p)$

f)
$$f_{X_1+X_2}(x) = \begin{cases} x, & 0 \le x < 1 \\ 2-x, & 1 \le x < 2 \\ 0, & \text{אחרת} \end{cases}$$

$$f_{X_1+X_2+X_3}(x) = \begin{cases} \frac{x^2}{2}, & 0 \le x \le 1 \\ -x^2+3x-\frac{3}{2}, & 1 \le x \le 2 \\ \frac{x^2}{2} - 3x + \frac{9}{2}, & 2 \le x \le 3 \\ 0, & \text{אחרת} \end{cases}$$

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4) a)
$$f_{X^2}(t) = \begin{cases} \frac{1}{3\sqrt{t}}, & 0 < t \le 1 \\ \frac{1}{6\sqrt{t}}, & 1 < t < 4 \\ 0, & \text{אחרת} \end{cases}$$

$EX^2 = VX + (EX)^2$

$EX^2 = \frac{3^2}{12} + \left(\frac{1}{2}\right)^2 = 1$

b) $X^2 \sim \Gamma(\frac{1}{2}, \frac{1}{2})$, $EX^2 = VX = 1$

c)
$$f_{X^2}(t) = \begin{cases} \frac{\lambda}{2\sqrt{t}} e^{-\lambda\sqrt{t}}, & t > 0 \\ 0, & t \le 0 \end{cases}$$

$EX^2 = VX + (EX)^2 = \frac{2}{\lambda^2}$

d) ~~$f_{X^2}(t) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \frac{1}{2\sqrt{t}} \cdot t^{\alpha-1} e^{-\lambda t}$, $t>0$~~

$EX^2 = \frac{\alpha}{\lambda^2} + \frac{\alpha^2}{\lambda^2} = \frac{\alpha(\alpha+1)}{\lambda^2}$

תרגיל 1 .5) a)

$$X_i \sim U(a,b) \qquad \bar{X} = \frac{1}{2}(X_1 + X_2)$$

$$f_{\bar{X}}(t) = \begin{cases} \dfrac{2(t-a)}{(b-a)^2}, & a \le t \le \dfrac{a+b}{2} \\ \dfrac{2(b-t)}{(b-a)^2}, & \dfrac{a+b}{2} \le t \le b \\ 0, & \text{אחרת} \end{cases}$$

b)

$$f_{\tilde{X}}(x) = \begin{cases} n\dfrac{(x-a)^{n-1}}{(b-a)^n}, & a \le x \le b \\ 0, & \text{אחרת} \end{cases}$$

$$f_{\underset{\sim}{X}}(x) = \begin{cases} n\dfrac{(b-x)^{n-1}}{(b-a)^n}, & a \le x \le b \\ 0, & \text{אחרת} \end{cases}$$

c).

$$E\tilde{X} = \frac{a+nb}{n+1} \qquad E\underset{\sim}{X} = \frac{na+b}{n+1}$$

.3) a).

$$f_{X+Y}(k) = P(X+Y=k) = \begin{cases} \dfrac{k-1}{N^2}, & 2 \le k \le N+1 \\ \dfrac{2N+1-k}{N^2}, & N+1 \le k \le 2N \end{cases}$$

$k = 2, 3, 4, \dots, 2N$

b).

$$f_{X+Y}(k) = P(X+Y=k) =$$

$$= \sum_{m=0}^{k} \frac{\lambda^m}{m!} e^{-\lambda} \frac{\mu^{k-m}}{(k-m)!} e^{-\mu} =$$

$$= \sum_{m=0}^{k} \binom{k}{m} \lambda^m \mu^{k-m} \frac{e^{-(\lambda+\mu)}}{k!} = \frac{(\lambda+\mu)^k}{k!} e^{-(\lambda+\mu)}$$

$k = 0, 1, 2, \dots \qquad X+Y \sim P(\lambda+\mu)$

(e. (3 תרגיל 1

$$F_{X+Y}(t)=\begin{cases}0\,, & t<-1\\ \dfrac{(t+1)^2}{12}\,, & -1\le t\le 1\\ \dfrac{t}{3}\,, & 1\le t<2\\ 1-\dfrac{(4-t)^2}{12}\,, & 2\le t<4\\ 1\,, & t\ge 4\end{cases}$$

$f_{X+Y}(t)=F'_{X+Y}(t)$

$t\neq -1,1,2,4$ כאלה

(d.

$$f_{X+Y}(t)=\int_0^t \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}\frac{\lambda^\beta}{\Gamma(\beta)}(t-x)^{\beta-1}e^{-\lambda(t-x)}dx=$$

$$=\frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\Gamma(\beta)}\cdot e^{-\lambda t}\int_0^t x^{\alpha-1}(t-x)^{\beta-1}dx=$$

$$=\frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\cdot\Gamma(\beta)}t^{\alpha+\beta-1}\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}dx=$$

$$=\frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha)\Gamma(\beta)}t^{\alpha+\beta-1}e^{-\lambda t}\cdot B(\alpha,\beta)$$

$$=\frac{\lambda^{\alpha+\beta}}{\Gamma(\alpha+\beta)}t^{\alpha+\beta-1}e^{-\lambda t}\,,\quad \underline{t>0}$$

$$X+Y\sim\Gamma(\alpha+\beta,\lambda)$$

(e.

$$f_{X+Y}(t)=\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}\,\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}\cdot\frac{1}{\sqrt{2\pi}\,\sigma_2}e^{-\frac{(t-x-\mu_2)^2}{2\sigma_2^2}}dx=\ldots$$

$$\ldots=\frac{1}{\sqrt{2\pi}\sqrt{\sigma_1^2+\sigma_2^2}}e^{-\frac{(t-\mu_1-\mu_2)^2}{2(\sigma_1^2+\sigma_2^2)}}$$

$$X+Y\sim N(\mu_1+\mu_2,\sigma_1^2+\sigma_2^2)$$