סטטיסטיקה מתמטית

תרגיל 10

1) מצא מבחן MP (בעל עוצמה מקסימלית) ברמת המובהקות מסויימת α, $0<\alpha<1$,
לבדיקת השערה פשוטה $H_0: \theta=\theta_0$
כנגד אלטרנטיבה פשוטה $H_1: \theta=\theta_1$
המבחן מבוסס על יחס נראות L_0/L_1
כאשר $L_1=L(x_1, \dots x_n, \theta_1)$, $L_0=L(x_1, \dots x_n, \theta_0)$

$$L(x_1, \dots x_n)=\prod_{i=1}^{n} f(x_i, \theta)$$

עבור התפלגויות $f(x, \theta)$ הבאות:

a). $N(\mu, \sigma^2)$ $\theta=\mu$, σ ידוע.

b). $Exp(\lambda)$, $\theta=\lambda$, $0<\lambda<\infty$

c). $N(\mu, \sigma^2)$, $\theta=\sigma$, μ ידוע. , $\sigma>0$

d). $Beta(a, 1)$, $\theta=a$, $0<a<+\infty$

e). $B(1, p)$, $\theta=p$, $0<p<1$

f). $G(p)$, $\theta=p$, $0<p<1$

g). $P(\lambda)$. $\theta=\lambda$, $0<\lambda<+\infty$

מצא גם את עוצמת המבחן $\pi(\theta_1)$
במקרה כאשר התפלגות בדידה (e), (f), (g).)
תאור את כל הערכים אפשריים של ר"מ α.

תרגיל 10

2). מצא מבחן UMP (בעל עוצמה מקסימלית במדה שוה) בר"מ α לבדיקת השערה

(i) $H_0: \theta \le \theta_0$ כנגד $H_1: \theta > \theta_0$

(ii) $H_0: \theta \ge \theta_0$ כנגד $H_1: \theta < \theta_1$

עבור התפלגויות: a), b), c), d) מתרגיל 1).

מצא את פונקצ"ית העוצמה $\pi(\theta)$

ואת ההסתברויות של טעויות מסוג I

ומסוג II, $\alpha(\theta)$, $\beta(\theta)$.

שרטט את הגרף של פ' העוצמה $\pi(\theta)$.

3). מצא מבחן דו-צדדי ברמת המובהקות α לבדיקת השערות: $H_0: \theta = \theta_0$ כנגד אלטרנטיבה $H: \theta \ne \theta_0$ ומצא את פונקצ"ית העוצמה $\pi(\theta)$,

עבור התפלגויות a) - d) מתרגיל 1).

4). $(X_1 \dots X_n)$ מדגם מהתפלגות אחידה $U(0,b)$.

$\tilde{X} = \max(X_1 \dots X_n)$!

מצא מבחן בר"מ α לבדיקת השערות

(i) $\begin{cases} H_0: b \ge b_0 \\ H_1: b < b_0 \end{cases}$, (ii) $\begin{cases} H_0: b \le b_0 \\ H_1: b > b_0 \end{cases}$

מבוסס על מ"מ $\tilde{X}$.

מצא את פונקצ"ית העוצמה $\pi(b)$ של המבחן.

$L = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n e$

$\frac{L_0}{L_1} = e^{-\frac{1}{2\sigma^2}}[$

$= e^{-\frac{1}{2\sigma^2}}[$

$C = \{(x_1 \dots x_n)$

$C = \{(x_1 \dots$

$\pi(\mu_0) = P_{\mu=}$

$= P_{\mu=\mu_0}\left(\frac{\bar{X} -}{\frac{\sigma}{\sqrt{n}}}\right.$

$\frac{x_2 - \mu_0}{\frac{\sigma}{\sqrt{n}}} = z_{1-}$

$C = \{$

$\pi(\mu_1) = P_{\mu=\mu_1}$

$= 1 - \Phi$

$= \Phi\left(\frac{\sqrt{n}}{\sigma}\right.$

$C = \{(x_1 \dots x_n):$

$\pi(\mu_1) = \Phi$

פתרונות $\boxed{\text{תרגיל 10}}$ (4)

(1 .(ב

$$L_1 = \lambda^n e^{-\lambda \Sigma x_i}$$

$$\frac{L_0}{L_1} = \left(\frac{\lambda_0}{\lambda_1}\right)^n e^{-(\lambda_0 - \lambda_1)\sum_{i=1}^{n} x_i} \le \gamma$$

$\underline{\lambda_0 < \lambda_1 \ (i)}$

$$C = \{(x_1 \ldots x_n) : \bar{x} \le \gamma_1\}$$

$$\pi(\lambda_0) = P_{\lambda=\lambda_0}(\bar{X} \le \gamma_1) = \alpha$$

$$\pi(\lambda_0) = P_{\lambda=\lambda_0}(2n\lambda_0 \bar{X} \le 2n\lambda_0\gamma_1) = \alpha \qquad \boxed{2n\lambda_0\bar{X} \sim \chi^2_{2n}}$$

$$2n\lambda_0\gamma_1 = \chi^2_{2n,\alpha}$$

$$C = \left\{(x_1 \ldots x_n) : \bar{x} \le \frac{\chi^2_{2n,\alpha}}{2n\lambda_0}\right\}$$

$$\pi(\lambda_1) = P_{\lambda=\lambda_1}\left(\bar{X} \le \frac{\chi^2_{2n,\alpha}}{2n\lambda_0}\right) = \qquad \boxed{2n\lambda_1\bar{X} \sim \chi^2_{2n}}$$

$$= P_{\lambda=\lambda_1}\left(2n\lambda_1\bar{X} \le \frac{\lambda_1}{\lambda_0}\chi^2_{2n,\alpha}\right) =$$

$$= F_{\chi^2_{2n}}\left(\frac{\lambda_1}{\lambda_0}\chi^2_{2n,\alpha}\right)$$

$\underline{\lambda_0 > \lambda_1 \ (ii)}$

$$C = \left\{(x_1 \ldots x_n) : \bar{x} \ge \frac{\chi^2_{2n,1-\alpha}}{2n\lambda_0}\right\}$$

$$\pi(\lambda_1) = P_{\lambda=\lambda_1}\left(\bar{X} \ge \frac{\chi^2_{2n,1-\alpha}}{2n\lambda_0}\right) =$$

$$= P_{\lambda=\lambda_1}\left(2n\lambda_1\bar{X} \ge \frac{\lambda_1}{\lambda_0}\chi^2_{2n,1-\alpha}\right) =$$

$$= 1 - F_{\chi^2_{2n,1-\alpha}}\left(\frac{\lambda_1}{\lambda_0}\chi^2_{2n,1-\alpha}\right)$$

פתרונות

תרגיל 10 ⑤

$$\frac{L_0}{L_1} = \frac{\sigma_1^n}{\sigma_0^n} e^{-\left(\frac{1}{2\sigma_0^2} - \frac{1}{2\sigma_1^2}\right)\sum_{i=1}^{n}(x_i-\mu)^2}$$ ג.

$$C = \left\{(x_1 \dots x_n) : \frac{L_0}{L_1} \le \gamma\right\}$$

(i) $\sigma_0 < \sigma_1$

$$C = \left\{(x_1 \dots x_n) : \sum_{i=1}^{n}(x_i-\mu)^2 \ge \gamma_1\right\}$$

$$\pi(\sigma_0) = P_{\sigma=\sigma_0}\left(\sum_{i=1}^{n}(X_i-\mu)^2 \ge \gamma_1\right) =$$

$$= 1 - P_{\sigma=\sigma_0}\left(\frac{1}{\sigma_0^2}\sum_{i=1}^{n}(X_i-\mu)^2 \le \frac{\gamma_1}{\sigma_0^2}\right) = \alpha$$

$$\boxed{\frac{1}{\sigma_0^2}\sum_{i=1}^{n}(X_i-\mu)^2 \sim \chi^2_n \quad \text{אם } \sigma=\sigma_0}$$

$$\frac{\gamma_1}{\sigma_0^2} = \chi^2_{n,1-\alpha}$$

$$\gamma_1 = \sigma_0^2\chi^2_{n,1-\alpha}$$

$$C = \left\{(x_1 \dots x_n) : \sum_{i=1}^{n}(x_i-\mu)^2 \ge \sigma_0^2\chi^2_{n,1-\alpha}\right\}$$

$$\pi(\sigma_1) = P_{\sigma=\sigma_1}\left(\sum_{i=1}^{n}(X_i-\mu)^2 \ge \sigma_0^2\chi^2_{n,1-\alpha}\right) =$$

$$= 1 - P_{\sigma=\sigma_1}\left(\frac{1}{\sigma_1^2}\sum_{i=1}^{n}(X_i-\mu)^2 \le \frac{\sigma_0^2}{\sigma_1^2}\chi^2_{n,1-\alpha}\right) =$$

$$= 1 - F_{\chi^2_n}\left(\frac{\sigma_0^2}{\sigma_1^2}\chi^2_{n,1-\alpha}\right)$$

(ii) $\sigma_0 > \sigma_1$

$$C = \left\{(x_1 \dots x_n) : \sum_{i=1}^{n}(x_i-\mu)^2 \le \sigma_0^2\chi^2_{n,\alpha}\right\}$$

$$\pi(\sigma_1) = F_{\chi^2_n}\left(\frac{\sigma_0^2}{\sigma_1^2}\chi^2_{n,\alpha}\right)$$

$L = p^{\sum_{i=1}^n x_i}$ (

$\frac{L_0}{L_1} = \left(\frac{u_0}{u_1}\right)$

$C = \{(x_1 \dots x_n):$

$C = \{(x_1 \dots x_n):$

$\pi(p_0) = P_{p=p}$

$= \sum_{k \le \gamma_2} ($

$= \sum_{k=0}^{m} \binom{n}{k} p_0^k$

$n = 1, 2, 3,$

$C_{m,n} = \{(x_1 \dots x$

$\pi(p_1) = P_{p=p_1}$

$C_{m,n} = \{(x_1 \dots$

$\alpha_{m,n} =$

$\pi(p_1) = \sum_{k=m}^{n}$

פתרונות | תרגיל 10 | 6

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$$L = a^n \left(\prod_{i=1}^n x_i\right)^{a-1} \qquad 0 \le x_i \le 1$$

$$\frac{L_0}{L_1} = \left(\frac{a_0}{a_1}\right)^n \left(\prod_{i=1}^n x_i\right)^{a_0 - a_1}$$

$$C = \left\{(x_1 \dots x_n) : \frac{L_0}{L_1} \le \gamma\right\}$$

(i) $a_1 > a_0$

$$C = \left\{(x_1 \dots x_n) : -\sum_{i=1}^n \ln x_i \le \gamma_1\right\}$$

$$\pi(a_0) = P_{a=a_0}\left(-\sum_{i=1}^n \ln X_i \le \gamma_1\right) = \alpha$$

$$\pi(a_0) = P_{a=a_0}\left(-2a_0 \sum_{i=1}^n \ln X_i \le 2a_0\gamma_1\right) \qquad -2a_0 \sum_{i=1}^n \ln X_i \sim \chi^2_{2n}$$

$$2a_0 \gamma_1 = \chi^2_{2n,\alpha}$$

$$C = \left\{(x_1 \dots x_n) : -\sum_{i=1}^n \ln x_i \le \frac{1}{2a_0}\chi^2_{2n,\alpha}\right\}$$

$$\pi(a_1) = P_{a=a_1}\left(-\sum_{i=1}^n \ln X_i \le \frac{1}{2a_0}\chi^2_{2n,\alpha}\right) =$$

$$= P_{a=a_1}\left(-2a_1 \sum_{i=1}^n \ln X_i \le \frac{a_1}{a_0}\chi^2_{2n,\alpha}\right) =$$

$$= F_{\chi^2_{2n}}\left(\frac{a_1}{a_0}\chi^2_{2n,\alpha}\right)$$

(ii) $a_1 < a_0$

$$C = \left\{(x_1 \dots x_n) : -\sum_{i=1}^n \ln x_i \ge \frac{\chi^2_{2n,1-\alpha}}{2a_0}\right\}$$

$$\pi(a_1) = 1 - F_{\chi^2_{2n}}\left(\frac{a_1}{a_0}\chi^2_{2n,1-\alpha}\right)$$

⑧ מרצ'ס 01 (פתרון)

1) .f) .

$$L = p^n (1-p)^{\sum_{i=1}^n x_i - n}$$

$$\frac{L_0}{L_1} = \frac{p_0^n (1-p_1)^n}{p_1^n (1-p_0)^n} \cdot \left(\frac{1-p_0}{1-p_1}\right)^{\sum_{i=1}^n x_i}$$

$$C = \left\{(x_1 \dots x_n) : \frac{L_0}{L_1} \le \gamma\right\} =$$

$$= \left\{(x_1 \dots x_n) : \sum_{i=1}^n x_i \ln \frac{1-p_0}{1-p_1} \le \gamma_1\right\}$$

(i) $p_0 < p_1$

$$\boxed{\frac{1-p_0}{1-p_1} > 1}$$

$$C = \left\{(x_1 \dots x_n) : \sum_{i=1}^n x_i \le \gamma_2\right\}$$

$$\sum_{i=1}^n X_i \sim NB(n, p)$$

$$\pi(p_0) = P_{p=p_0}\left(\sum_{i=1}^n X_i \le \gamma_2\right) =$$

$m = [\gamma_2]$

$$= \sum_{k=n}^m \binom{k-1}{n-1} p_0^n (1-p_0)^{k-n} = \alpha_{m,n}$$

לכל $n = 1,2,\dots$ ולכל $m = n, n+1, \dots$

מבחן $C_{m,n} = \left\{(x_1 \dots x_n) : \sum_{i=1}^n x_i \le m\right\}$

בעל כ"מ $\alpha_{m,n}$ ועוצמה

$$\pi(p_1) = P_{p=p_1}\left(\sum_{i=1}^n X_i \le m\right) = \sum_{k=n}^m \binom{k-1}{n-1} p_1^n (1-p_1)^{k-n}$$

(ii) $p_0 > p_1$

לכל $n = 1,2,\dots$ ולכל

מבחן $C_{m,n} = \left\{(x_1 \dots x_n) : \sum_{i=1}^n x_i \ge m\right\}$

בעל ר"מ $\alpha_{m,n} = \sum_{k=m}^\infty \binom{k-1}{n-1} p_0^n (1-p_0)^{k-n}$

ועוצמה

$$\pi(p_1) = \sum_{k=m}^\infty \binom{k-1}{n-1} p_1^n (1-p_1)^{k-n}$$

$\boxed{\text{תרגיל 10}}$ פתרונות

2. a. (i)

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \begin{cases} H_0: \mu \le \mu_0 \\ H_1: \mu > \mu_0 \end{cases}$$

$$C = \left\{ (x_1 \dots x_n) : \ \bar{x} \geq \mu_0 + \frac{\sigma}{\sqrt{n}} z_{1-\alpha} \right\}$$

$$\pi(\mu) = \Phi\left(\frac{\sqrt{n}}{\sigma}(\mu - \mu_0) - z_{1-\alpha} \right)$$ לכל μ

כ"א $$\sup_{\mu \le \mu_0} \alpha(\mu) = \pi(\mu_0) = \Phi(-z_{1-\alpha}) = \alpha$$

לכל $\mu_0 < \mu_1$ C מבחן MP ל- $\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu = \mu_1 \end{cases}$

C מבחן UMP

(ii) $\begin{cases} H_0: \mu \geq \mu_0 \\ H_1: \mu < \mu_0 \end{cases}$

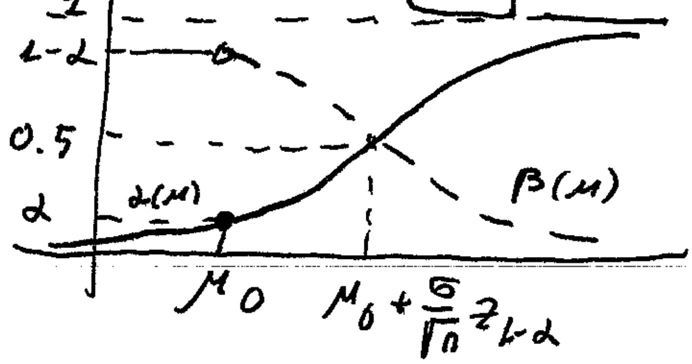
$$C = \left\{ (x_1 \dots x_n) : \ \bar{x} \le \mu_0 - \frac{\sigma}{\sqrt{n}} z_{1-\alpha} \right\}$$

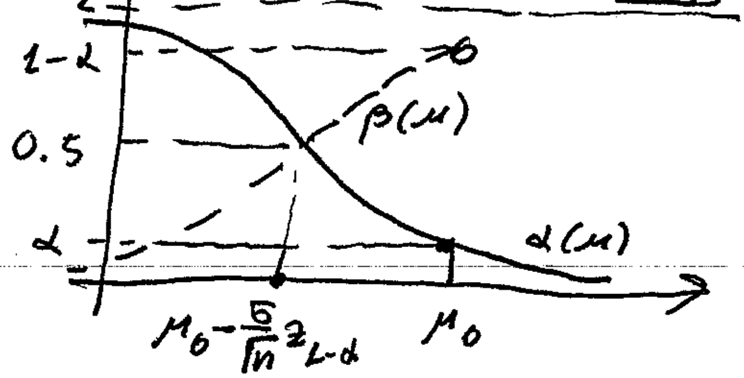
$$\pi(\mu) = \Phi\left(\frac{\sqrt{n}}{\sigma}(\mu_0 - \mu) - z_{1-\alpha} \right)$$ לכל μ

כ"א $$\sup_{\mu \geq \mu_0} \alpha(\mu) = \pi(\mu_0) = \Phi(-z_{1-\alpha}) = \alpha$$

לכל $\mu_0 > \mu_1$ C מבחן MP ל- $\begin{cases} H_0: \mu = \mu_0 \\ H_1: \mu = \mu_1 \end{cases}$

$\Leftarrow$ C מבחן UMP





פתרונות

תרגיל 10

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2. b. (i)

$$\begin{cases} H_0: \lambda \le \lambda_0 \\ H_1: \lambda > \lambda_0 \end{cases}$$ מבחן UMP

$$C = \left\{ (x_1 \dots x_n) : \bar{x} \le \frac{\chi^2_{2n,\alpha}}{2n\lambda_0} \right\}$$

$$\pi(\lambda) = F_{\chi^2_{2n}}\left(\frac{\lambda}{\lambda_0} \chi^2_{2n,\alpha} \right)$$

$$\sup_{\lambda \le \lambda_0} F_{\chi^2_{2n}}\left(\frac{\lambda}{\lambda_0} \chi^2_{2n,\alpha} \right) = \alpha$$ ר"מ

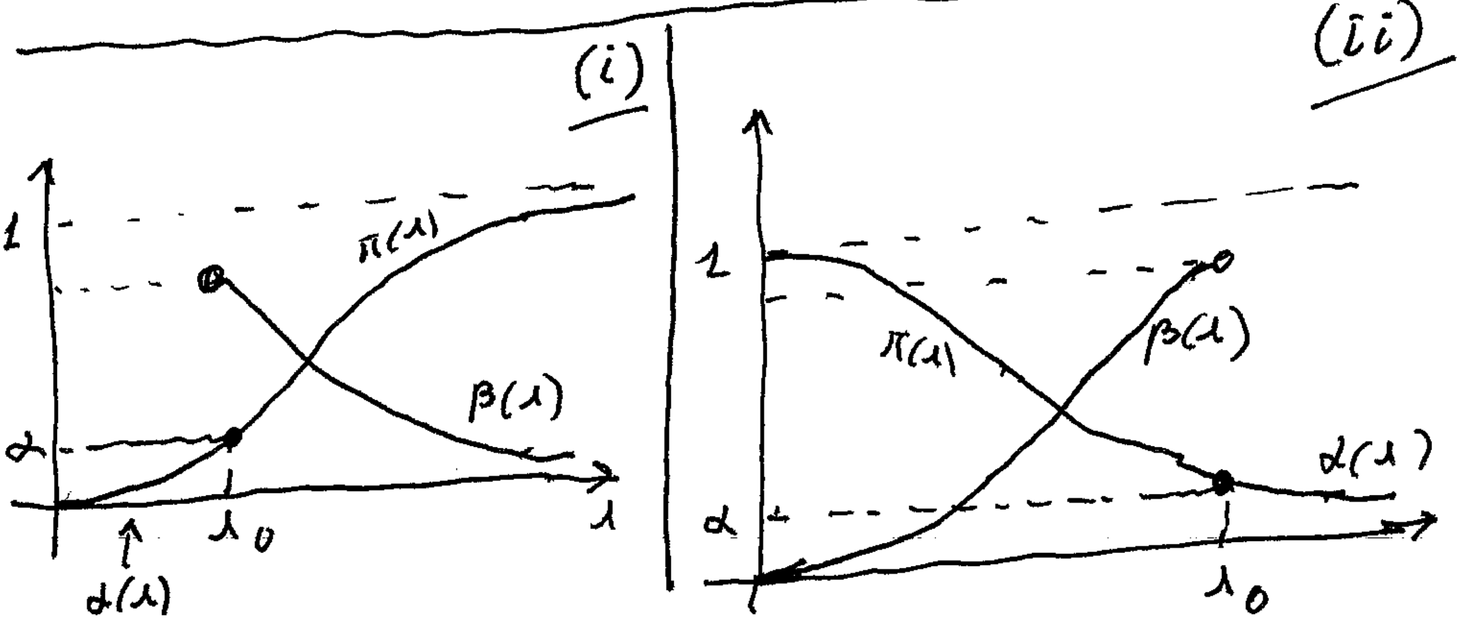
(ii)

$$\begin{cases} H_0: \lambda \ge \lambda_0 \\ H_1: \lambda < \lambda_0 \end{cases}$$ מבחן UMP

$$C = \left\{ (x_1 \dots x_n) : \bar{x} \ge \frac{\chi^2_{2n,1-\alpha}}{2n\lambda_0} \right\}$$

$$\pi(\lambda) = 1 - F_{\chi^2_{2n}}\left(\frac{\lambda}{\lambda_0} \chi^2_{2n,1-\alpha} \right)$$

$$\sup_{\lambda \ge \lambda_0} \pi(\lambda) = 1 - F_{\chi^2_{2n}}\left(\chi^2_{2n,1-\alpha} \right) = \alpha$$ ר"מ



(12) תרגיל 10

פתרונות

2) א) (i)

מבחן UMP
$$\begin{cases} H_0: \sigma \le \sigma_0 \\ H_1: \sigma > \sigma_1 \end{cases}$$

$$C = \left\{ (x_1 \dots x_n) : \sum_{i=1}^{n} (x_i - \mu)^2 \ge \sigma_0^2 \chi^2_{n,1-\alpha} \right\}$$

$$\pi(\sigma) = 1 - F_{\chi^2_n}\left(\frac{\sigma_0^2}{\sigma^2} \chi^2_{n,1-\alpha} \right)$$

$$\sup_{\sigma \le \sigma_0} \pi(\sigma) = \pi(\sigma_0) = 1 - F_{\chi^2_n}\left(\chi^2_{n,1-\alpha}\right) = \alpha$$ כנ"ל

(ii)

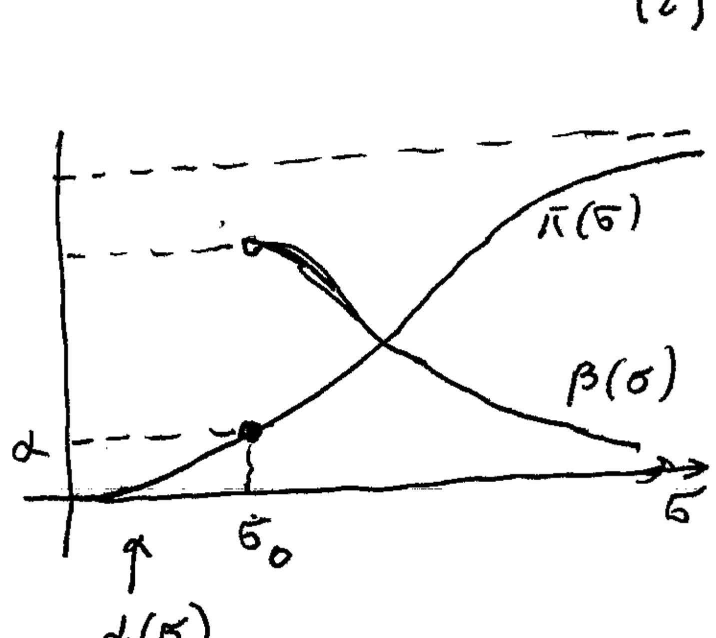
מבחן UMP
$$\begin{cases} H_0: \sigma \ge \sigma_0 \\ H_1: \sigma \;\; \sigma \end{cases}$$

$$C = \left\{ (x_1 \dots x_n) : \sum_{i=1}^{n} (x_i - \mu)^2 \le \sigma_0^2 \chi^2_{n,\alpha} \right\}$$

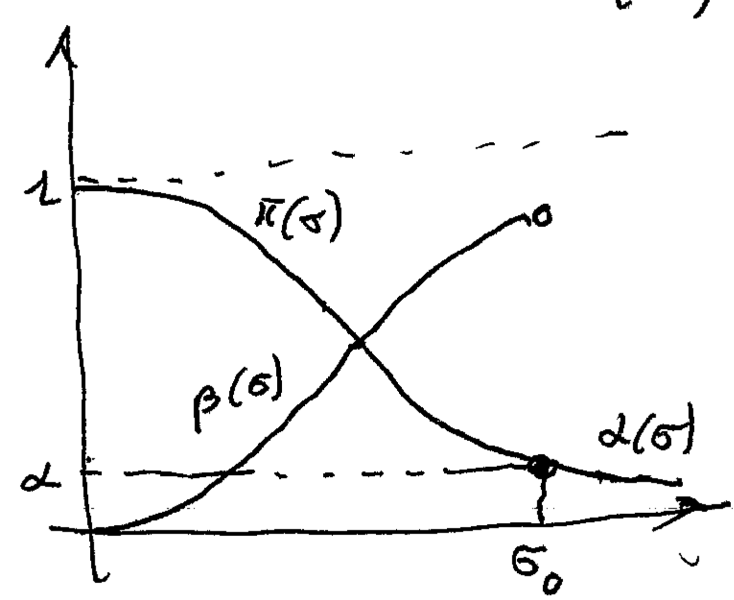
$$\sup_{\sigma \ge \sigma_0} \pi(\sigma) = \pi(\sigma_0) = F_{\chi^2_n}\left(\chi^2_{n,\alpha}\right) = \alpha$$ כנ"ל

$$\pi(\sigma) = F_{\chi^2_n}\left(\frac{\sigma_0^2}{\sigma^2} \chi^2_{n,\alpha} \right)$$ כללית

(i)



(ii)



פתרונות

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$0 < a < \infty$

2. .d (i)

$$\begin{cases} H_0 : a \le a_0 \\ H_1 : a > a_0 \end{cases}$$

מבחן UMP

$$C = \left\{ (x_1 \dots x_n) : -\sum_{i=1}^{n} \ln x_i \le \frac{\chi^2_{2n,\alpha}}{2a_0} \right\}$$

$$\pi(a) = F_{\chi^2_{2n}} \left(\frac{a}{a_0} \chi^2_{2n,\alpha} \right)$$

$$\sup_{a \le a_0} \pi(a) = \pi(a_0) = F_{\chi^2_{2n}} \left(\chi^2_{2n,\alpha} \right) \overset{\text{כ״מ}}{=} \alpha$$

(ii)

$$\begin{cases} H_0 : a \ge a_0 \\ H_1 : a < a_0 \end{cases}$$

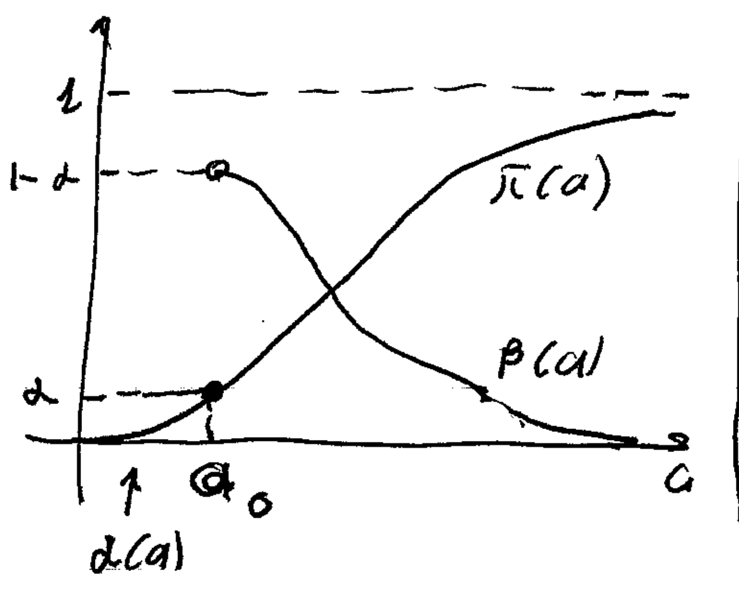
מבחן UMP

$$C = \left\{ (x_1 \dots x_n) : -\sum_{i=1}^{n} \ln x_i \ge \frac{\chi^2_{2n,1-\alpha}}{2a_0} \right\}$$

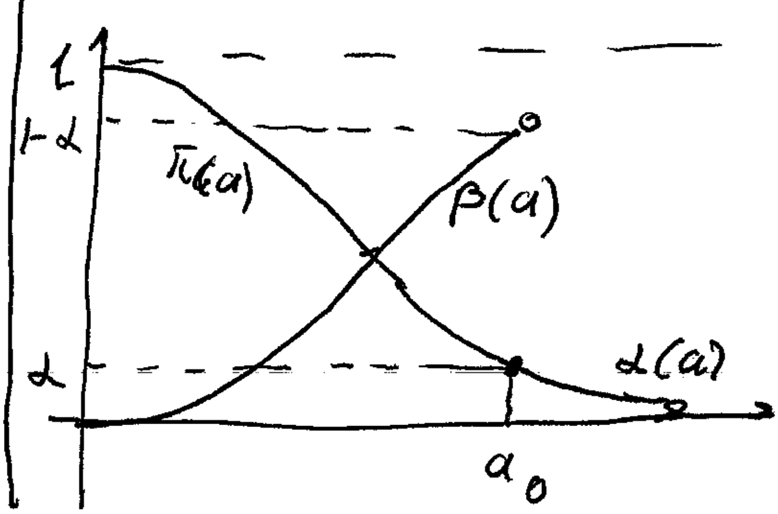
$$\pi(a) = 1 - F_{\chi^2_{2n}} \left(\frac{a}{a_0} \chi^2_{2n,1-\alpha} \right)$$

$$\sup_{a \ge a_0} \pi(a) = \pi(a_0) = 1 - F_{\chi^2_{2n}} \left(\chi^2_{2n,1-\alpha} \right) \overset{\text{כ״מ}}{=} \alpha$$

(i)



(ii)



פתרונות קרג'סון (14)

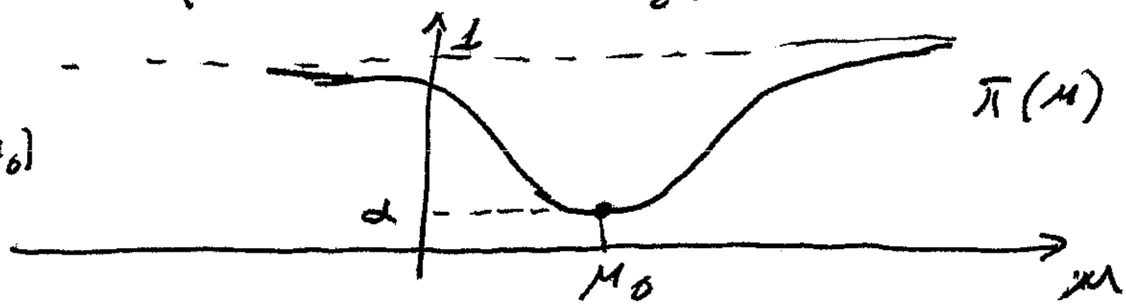
3. a.

ר״מ α $\quad \begin{cases} H_0 : \mu = \mu_0 \\ H_1 : \mu \neq \mu_0 \end{cases}$

$$C = \left\{ (x_1 \dots x_n) : |\bar{x} - \mu_0| \geq \frac{\sigma}{\sqrt{n}} z_{1-\frac{\alpha}{2}} \right\}$$

$$\pi(\mu) = \Phi\left(\frac{\sqrt{n}}{\sigma}(\mu_0 - \mu) - z_{1-\frac{\alpha}{2}}\right) + \Phi\left(\frac{\sqrt{n}}{\sigma}(\mu - \mu_0) - z_{1-\frac{\alpha}{2}}\right)$$

$\alpha = \pi(\mu_0)$

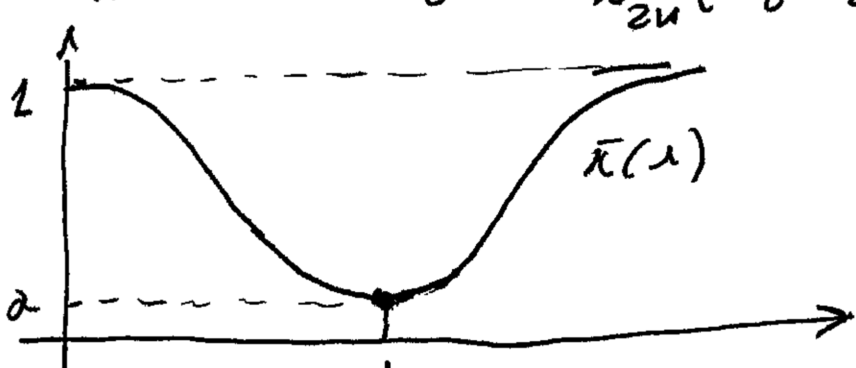


b.

ר״מ α, $\quad \begin{cases} H_0 : \lambda = \lambda_0 \\ H_1 : \lambda \neq \lambda_0 \end{cases}$

$$C = \left\{ (x_1 \dots x_n) : \bar{x} \geq \frac{\chi^2_{2n, 1-\frac{\alpha}{2}}}{2n\lambda_0} \text{ או } \bar{x} \leq \frac{\chi^2_{2n, \alpha/2}}{2n\lambda_0} \right\}$$

$$\pi(\lambda) = 1 - F_{\chi^2_{2n}}\left(\frac{\lambda}{\lambda_0} \chi^2_{2n, 1-\frac{\alpha}{2}}\right) + F_{\chi^2_{2n}}\left(\frac{\lambda}{\lambda_0} \chi^2_{2n, \frac{\alpha}{2}}\right)$$



$$\pi(\lambda_0) = 1 - F_{\chi^2_{2n}}\left(\chi^2_{2n, 1-\frac{\alpha}{2}}\right) + F_{\chi^2_{2n}}\left(\chi^2_{2n, \frac{\alpha}{2}}\right) =$$

$$= 1 - \left(1 - \frac{\alpha}{2}\right) + \frac{\alpha}{2} = \alpha \qquad \text{ר״מ}$$

פתרונות

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3. c.

$$\begin{cases} H_0: \sigma=\sigma_0 \\ H_1: \sigma\neq\sigma_0 \end{cases}$$

$$C=\left\{(x_1\ldots x_n):\ \sum_{i=1}^{n}(x_i-\mu)^2\geq\sigma_0^2\chi^2_{n,1-\frac{\alpha}{2}},\right.$$

$$\left.\text{או}\quad \sum_{i=1}^{n}(x_i-\mu)^2\leq\sigma_0^2\chi^2_{n,\frac{\alpha}{2}}.\right\}$$

$$\pi(\sigma)=1-F_{\chi^2_n}\left(\frac{\sigma_0^2}{\sigma^2}\chi^2_{n,1-\frac{\alpha}{2}}\right)+F_{\chi^2_n}\left(\frac{\sigma_0^2}{\sigma^2}\chi^2_{n,\frac{\alpha}{2}}\right)$$

$\pi(\sigma_0)=\alpha$ ר"ל

$$=1-F_{\chi^2_n}\left(\chi^2_{n,1-\frac{\alpha}{2}}\right)+F_{\chi^2_n}\left(\chi^2_{n,\frac{\alpha}{2}}\right)=$$

$$=1-\left(1-\frac{\alpha}{2}\right)+\frac{\alpha}{2}=\alpha$$

d.

$$\begin{cases} H_0: a=a_0 \\ H_1: a\neq a_0 \end{cases}$$

$$C=\left\{(x_1\ldots x_n):\ -\sum_{i=1}^{n}\ln x_i\leq\frac{\chi^2_{2n,\alpha/2}}{2a_0},\right.$$

$$\left.\text{או}\quad -\sum_{i=1}^{n}\ln x_i\geq\frac{\chi^2_{2n,1-\frac{\alpha}{2}}}{2a_0}\right\}$$

$$\pi(a)=1-F_{\chi^2_{2n}}\left(\frac{a}{a_0}\chi^2_{2n,1-\frac{\alpha}{2}}\right)+$$

$$+F_{\chi^2_{2n}}\left(\frac{a}{a_0}\chi^2_{2n,\frac{\alpha}{2}}\right)$$

$$\pi(a_0)=1-F_{\chi^2_{2n}}\left(\chi^2_{2n,1-\frac{\alpha}{2}}\right)+F_{\chi^2_{2n}}\left(\chi^2_{2n,\frac{\alpha}{2}}\right)=$$

ר"ל

$$=1-\left(1-\quad\right)+\frac{\alpha}{2}=\alpha$$

פתרון

תרגיל 10 ⑯

$\tilde{x} = \max_{1 \le i \le n} x_i$

$$\begin{cases} H_0: \theta \geq \theta_0 \\ H_1: \theta < \theta_0 \end{cases} \quad (i) \quad 4)$$

$\tilde{X} = \max_{1 \le i \le n} X_i$

לכל θ $\quad F_{\tilde{X}}(u) = \begin{cases} 0, & u < 0 \\ \left(\frac{u}{\theta}\right)^n, & 0 \le u < \theta \\ 1, & u \geq \theta \end{cases}$

$C = \{(x_1 \dots x_n) : \tilde{x} \le \gamma\}$

רוצים לקבל

$\pi(\theta_0) = P_{\theta = \theta_0}(\tilde{X} \le \gamma) = \alpha$

ולכל $\theta \geq \theta_0$

$\pi(\theta) \le \alpha$

$\Longleftarrow$ אם $\theta = \theta_0$

$\alpha = F_{\tilde{X}}(\gamma)$

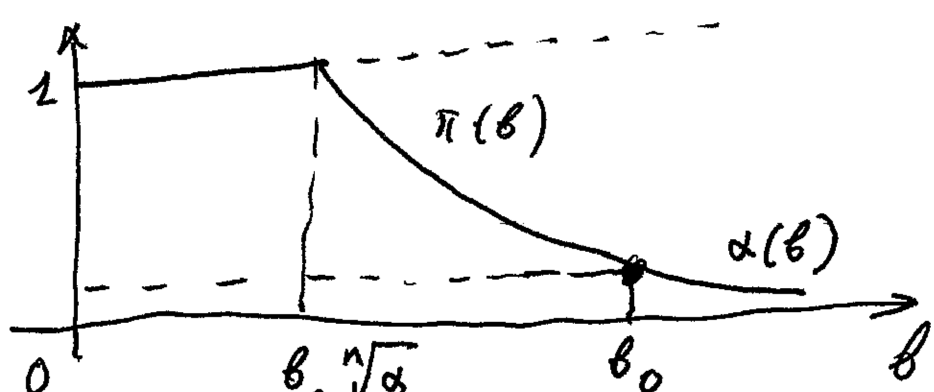
$\left(\frac{\gamma}{\theta_0}\right)^n = \alpha \Longrightarrow \gamma = \theta_0 \sqrt[n]{\alpha}$

$C = \{(x_1 \dots x_n) : \tilde{x} \le \theta_0 \sqrt[n]{\alpha}\}$

$\pi(\theta) = P_\theta(\tilde{X} \le \theta_0 \sqrt[n]{\alpha}) =$

$$= \begin{cases} 1, & \theta \le \sqrt[n]{\alpha}\,\theta_0 \\ \alpha \cdot \frac{\theta_0^n}{\theta^n}, & \theta \geq \sqrt[n]{\alpha} \cdot \theta_0 \end{cases}$$

$\sup_{\theta \geq \theta_0} \pi(\theta) = \pi(\theta_0) = \alpha$ כ״א



תרגיל 10

4. (ii)

$$\begin{cases} H_0: \theta \le \theta_0 \\ H_1: \theta > \theta_0 \end{cases}$$

$$C = \{(x_1 \dots x_n) : \tilde{x} \geq \gamma\}$$

$$\alpha = P_{\theta=\theta_0}(\tilde{X} \geq \gamma) = 1 - \left(\frac{\gamma}{\theta_0}\right)^n$$

$$\gamma = \theta_0 \sqrt[n]{1-\alpha}$$

$$C = \{(x_1 \dots x_n) : \tilde{x} \geq \theta_0 \sqrt[n]{1-\alpha}\}$$

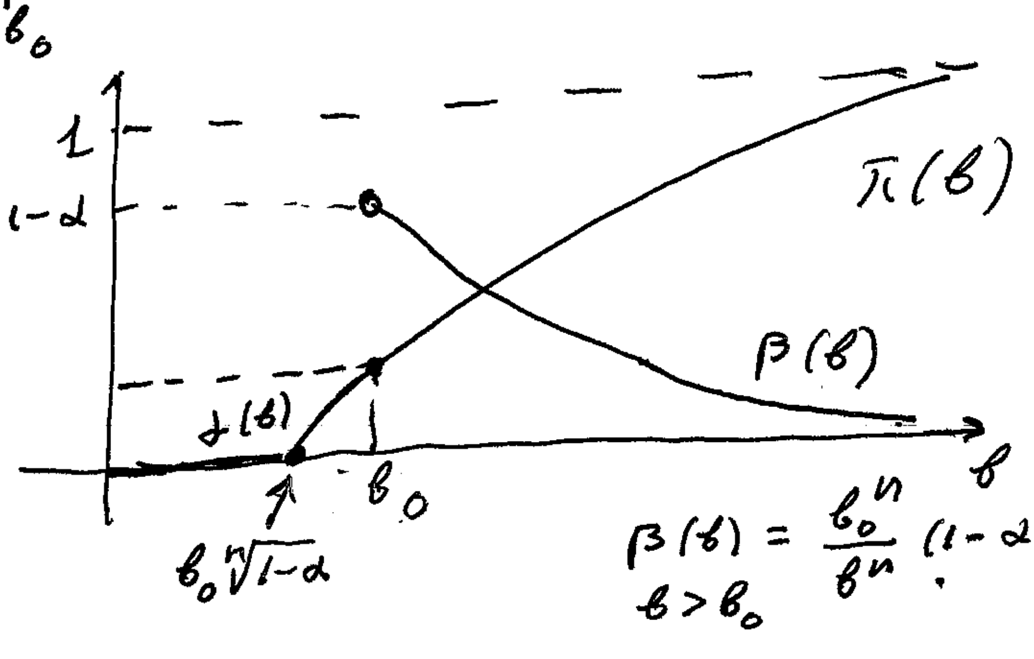
$$\pi(\theta) = P_\theta(\tilde{X} \geq \theta_0 \sqrt[n]{1-\alpha}) =$$

$$= 1 - F_{\tilde{X}}(\theta_0 \sqrt[n]{1-\alpha})$$

$$\pi(\theta) = \begin{cases} 0 , & \theta < \theta_0 \sqrt[n]{1-\alpha} \\ 1 - \frac{\theta_0^n}{\theta^n}(1-\alpha) , & \theta \geq \theta_0 \sqrt[n]{1-\alpha} \end{cases}$$

ר״א

$$\sup_{\theta \le \theta_0} \pi(\theta) = \pi(\theta_0) = \alpha$$



$$\beta(\theta) = \frac{\theta_0^n}{\theta^n}(1-\alpha) , \quad \theta > \theta_0$$