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The Tower of Hanoi with Forbidden Moves on Graphs with 3 Vertices

1. Abstract

The Tower of Hanoi problem, composed over a hundred years ago by Lucas [1], is stated as follows: Given are 3 pegs and a certain number n of disks of distinct sizes. Initially all disks are stacked (the tower) on the first peg (the source) ordered by size, with the smallest at the top and the largest at the bottom. The goal is to transfer them to the third peg (the destination), while obeying the following rules:

1. At each step only one disk can be moved;
2. The moved disk must be a topmost one;
3. At any moment, a disk cannot reside on a smaller one.

We consider variants of the classical 3-peg Tower of Hanoi problem, where limitations on the possible moves among the pegs are imposed. Each variant corresponds to a di-graph whose vertices are the pegs, and an edge from one vertex to another designates the ability of moving a disk from the first peg to the other, provided that the rules concerning the disk sizes are obeyed. There are 5 non-isomorphic graphs on 3 vertices, which are strongly connected - a necessary and sufficient condition for the existence of a solution to the problem. We will show all the cases, and solve some of them fully using algebraic tools. Then we will present a unified optimal algorithm for all these graphs, and prove its correctness and minimality.

References

1. Lucas É., "Récréations Mathématiques", *vol. III*, Gauthier-Villars, Paris, 1893.