Several Graph Layout Problems for Grids

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Abstract

A large number of theoretical and practical problems in various areas may be formulated as graph layout problems. Such problems arise in connection with planar graphs, the optimization of networks for parallel computer architectures, VLSI circuit design, and numerous other problems. Many interesting graph layout problems are NP-hard, and thus a lot of work has been done on solving them for some structured graph families.

Here we concentrate on the *minimal bandwidth linear arrangement* (bandwidth) problem, which may be posed in the following form.

For a graph $G = (V, E)$ with $|V| = n$, and a placement of its vertices at positions $1, 2, \ldots, n$ on the real line, the *length* of an edge is the distance between the positions of its vertices.

Problem. Given a graph $G = (V, E)$, find a placing of the vertices for which the maximal edge length is as small as possible. More formally we look for a vertex enumeration function $f : V \to \{1, \ldots, n\}$ such that

$$\eta(G, f) = \max \{|f(v) - f(u)| : (u, v) \in E\}$$

is minimal over all such enumerations:

$$MinBW(G) = \min_{f:V \to \{1,\ldots,n\}} \eta(G, f).$$

We find the minimal bandwidth value for torodial rectangular grids and present an algorithm for finding the corresponding optimal ordering. We also present a proof of the formula for the bandwidth value for regular grids, which is somewhat shorter than the original proof.