Combinatorial Dominance Analysis

Yochai Twitto

Department of Computer Science,
Ben-Gurion University of the Negev,
Be'er-Sheva 84105, Israel

January 13, 2004

Abstract

Combinatorial optimization problems are of great importance both theoretically and for studying real-world problems. A lot of research has been done on this kind of problems. In principle, we want an algorithm finding the best solution for a given problem, but sometimes it is hard and we are satisfied with an *approximation algorithm*. Approximation algorithms are important since some problems are too complex to be handled by a computer. Their goal is to give us an as good as possible solution, which can be computed efficiently.

A measure of *quality* of an algorithm enables us to compare and evaluate algorithms. For exact algorithms we compare mainly running time, but for approximation algorithms we first check the solution quality. Regarding the solution quality, we may analyze the worst case, mean case, typical case, etc. of an algorithm. There are several guarantees that an algorithm may give, for example:

- Approximation ratio – my solution is at least half (say) as good as the best solution.
- Probabilistic guarantee – my solution is optimal with high probability.
- Tradeoff behavior – tell me what error level you are willing to tolerate, and you will get a solution not exceeding this level. However, a higher accuracy level will cost you more.

The talk will concentrate on a much less common way of evaluating algorithms, i.e. according to the measure of *combinatorial dominance* [1, 3]. An example of a combinatorial dominance guarantee is: "The solution provided by this algorithm is always in the top 10% of all solutions of the problem". We will motivate such kind of analysis and compare it to the classical *approximation ratio* analysis.

As a part of the talk, we will overview and analyze some fundamental NP-hard problems for combinatorial dominance guarantees. Among the problems we shall see there are the travelling salesman problem, the minimum partition problem, maximum cut and more...

References

[1] D. Berend and S.S. Skiena. Combinatorial Dominance Guarantees for Heuristic Algorithms. Preprint.

[2] S.S. Skiena. Combinatorial Dominance and Heuristic Search. PDF presentation. http://www.cs.sunysb.edu/~skiena.

[3] G. Gutin, A. Vainshtein and A. Yeo. Domination Analysis of Combinatorial Optimization Problems. *Discrete Applied Mathematics* 129 (2003) 513-520.