CONVEX HULLS OF SETS IN THE PLANE

The convex hull of a set of points is the smallest convex set containing the points. The convex hull is a fundamental concept in mathematics and computational geometry. Other problems can be reduced to finding the convex hull – for example, halfspace intersection, Delaunay triangulation, Voronoi diagram, etc. An algorithm known as Graham scan [6] achieves an $O(n \log n)$ running time. This algorithm is optimal in the worst case. Another algorithm [7] for the same problem runs in $O(nh)$ time, where h is the number of hull points, and this outperforms the preceding algorithm if h happens to be very small. Kirkpatrick and Seidel [8] desined later an algorithm with $O(n \log h)$ runtime, which is always at least as good as the better of the above two algorithms. A simplification of their algorithm has been recently reported by Chan, Snocyink and Yap [2].

For some special sets of points, it is possible to improve the results above. We concentrate on the case when the set consists of all intersection points of n lines. A straightforward application of the algorithms above leads to a runtime of slightly more than $O(n^2)$. Attalah [1] and Ching and Lee [3] independently presented $O(n \log n)$ runtime worst-case algorithms with $O(n)$ space. Ching and Lee [3] also showed that this result is best possible. Devroye and Toussaint [4] and Golin, Langerman and Steiger [5] studied the case when the lines are random with certain distributions. In both models, they presented algorithms with linear expected time.

References

[1] M. Atallah. Computing the convex hull of line intersections, *J. Algorithms* **7** (1986), 285–288.

[2] T.M. Chan, J. Snocyink and C. K. Yap. Output-sensitive construction of polytopes in four dimensions and clipped Voronoi diagram in three, *Proc. 6th ACM-SIAM Sympos. on Discrete Algorithms* (1985), 282–291.

[3] Y.T. Ching and D.T. Lee, Finding the diameter of a set of lines,*Pattern Recognition* **18** (1985), 249–255.

[4] L. Devroye and G. Toussaint, Convex hulls for random lines, *J. Algorithms* **14** (1993), 381–394.

[5] M. Golin, S. Langerman and W. Steiger. The Convex Hull for Random Lines in the Plane, Proceedings of the Japan Conference on Discrete and Computational Geometry (JCDCG 2002), (2002)

[6] R.L Graham. An efficient algorithm for determining the convex hull of a finite planar set, *Inform. Process. Lett.* **1** 1972, 132-133.

[7] R.A. Jarvis. On the identification of the convex hull of a finite set of points in the plane.*Inform. Process. Lett.* **2** 1973, 18-21.

[8] D.G Kirkpatrick and R. Seidel. The ultimate planar convex hull algorithm?, *SIAM J. Comput.* **15** 1986, 287-299.