

# The Cyclic Multi-peg Tower of Hanoi

## Abstract

Variants of the classical Tower of Hanoi problem evolved in various directions. Two of these are allowing more than 3 pegs, and imposing limitations on the possible moves among pegs. Unlike the original problem, where the complexity is exponential, the unrestricted multi-peg problem for  $h \geq 4$  pegs is known to be sub-exponential. A natural question is whether this is the case even when we disallow direct moves between some pairs of pegs. In other words, the pegs are the vertices of some directed graph. In such a representation, the unrestricted problem corresponds to the complete graph  $K_h$  on  $h$  vertices.

In this talk, the  $\text{Cyclic}_h$  variant, where the pegs are arranged in a circle, and moves are allowed only from a peg to its successor (in the clockwise direction), is examined.

It turns out that the complexity of this variant grows exponentially as a function of the number of disks. Moreover, it is shown to behave regularly. Explicit lower and upper bounds for it for any  $h$  will be presented, together with an approach that lets this complexity be estimated arbitrarily well for any specific  $h$ . It will also be shown that the task becomes easier as the number of pegs grows.