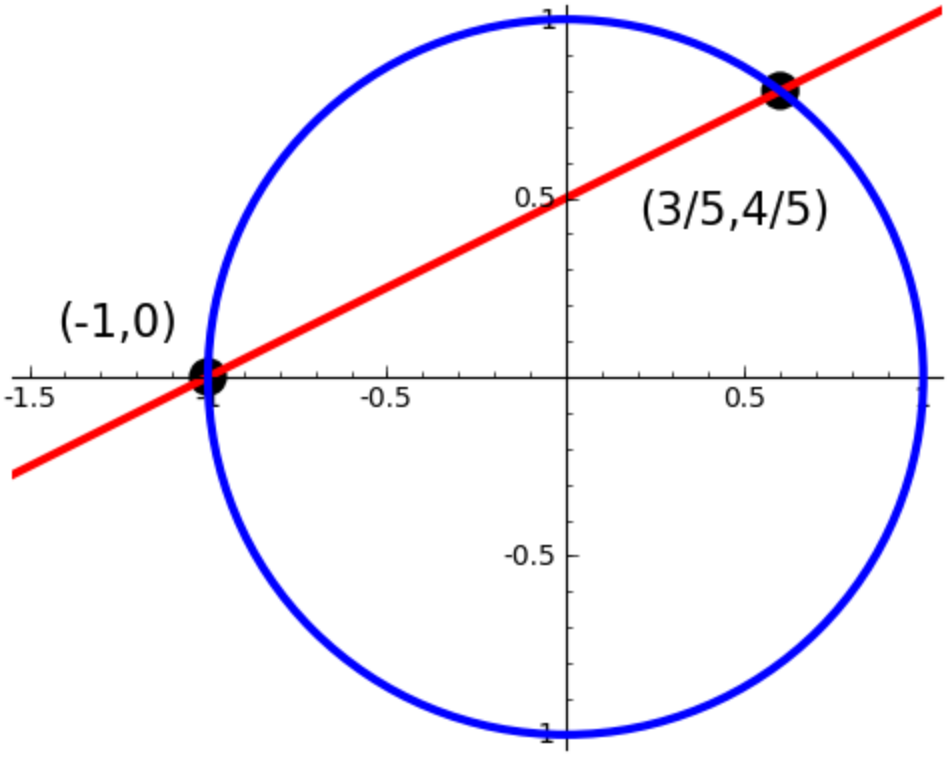
BSD-lecture

הדגמה של מציאת נקודות רציונליות חדשות על העקום

$$x^2 + y^2 = 1$$

```
G = circle((0,0),1, rgbcolor='blue', thickness=3)
G += point([(-1,0), (3/5,4/5)], pointsize=150, rgbcolor='black')
G += line([(-1-1,0-1/2), (3/5+1,4/5+1/2)], rgbcolor='red',
thickness=3)
G += text("(3/5,4/5)", (3/5-1/8,4/5-1/3),
rgbcolor='black',fontsize=16)
G += text("(-1,0)", (-1.25,0.15), rgbcolor='black',fontsize=16)
G.show(aspect_ratio=1,xmin=-1.5,xmax=1, ymin=-1,ymax=1)
```



הדגמה נוספת של מציאת נקודות על אותו עקום

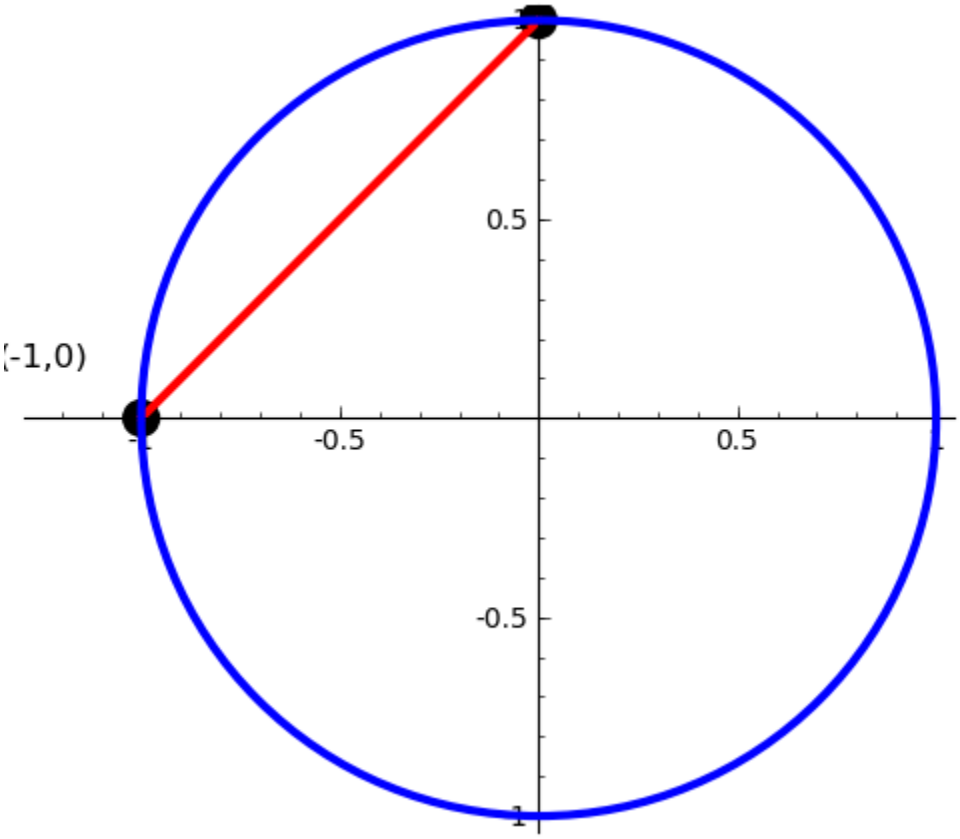
```
%auto
```

```
@interact
def __(t=('slope',tuple([1/n for n in [1..20]]))):
    t = QQ(t)
    x = (1-t^2)/(1+t^2)
    y = 2*t/(1+t^2)
    r = t.numerator()
    s = t.denominator()
    a = s^2 - r^2; b = 2*r*s; c = s^2 + r^2
    html('<h1 align=center>Point (x,y) = $%s$'%latex((x,y)))
    html('Pythagorean (a,b,c) = $%s$</h1>'%latex((a,b,c)))
    G = circle((0,0),1, rgbcolor='blue', thickness=3)
    G += point([(-1,0), (x,y)], pointsize=150, rgbcolor='black')
    G += line([(-1,0), (x,y)], rgbcolor='red', thickness=3)
    G += text("(-1,0)", (-1.25,0.15), rgbcolor='black',fontsize=12)
    try:
       G.save('a.png',aspect_ratio=1)
    except RuntimeError, msg:
        print msg
    html('<img src="cell://a.png">')
```

slope 1

Point (x,y) = $(0, 1)$

Pythagorean (a,b,c) = $(0, 2, 2$



נדגים כמה מהיכולות של

Sage

בעקומים אליפטיים ופונקציות ה

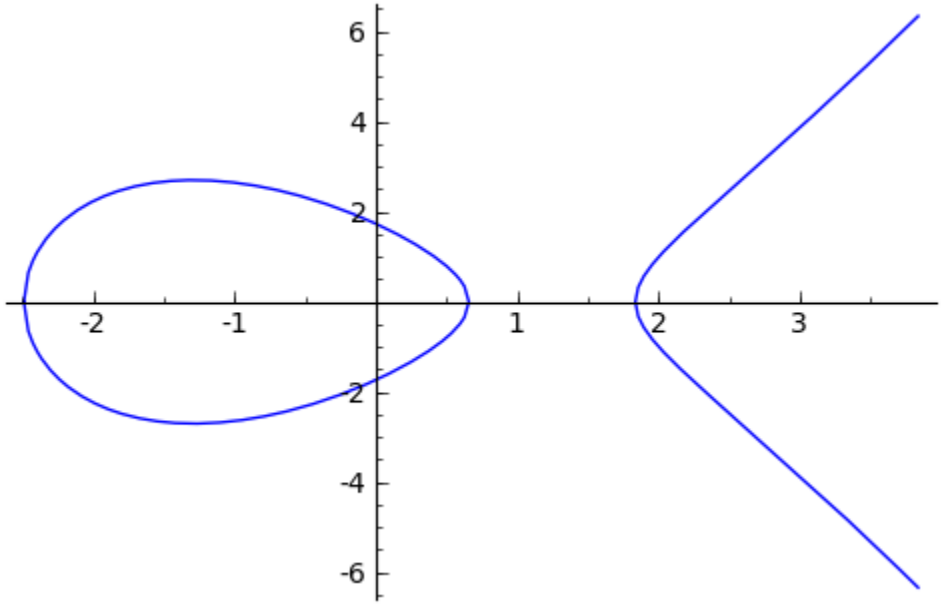
L

שלהן

```
E0=EllipticCurve([-5,3]); E0
```

```
Elliptic Curve defined by y^2 = x^3 - 5*x + 3 over Rational Field
```

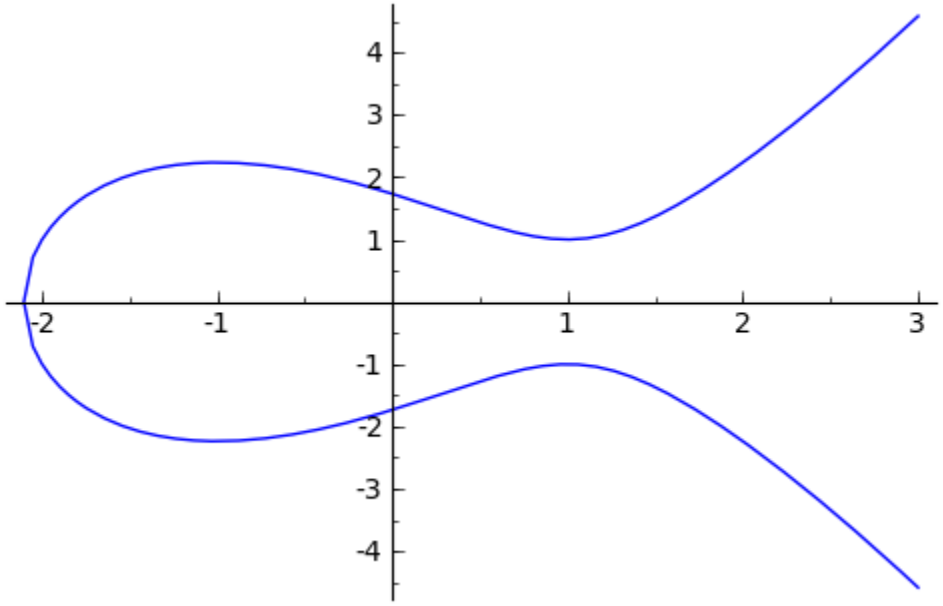
```
E0.plot(-3,3)
```




```
E1=EllipticCurve([-3,3]); E1
```

```
Elliptic Curve defined by y^2 = x^3 - 3*x + 3 over Rational Field
```

```
E1.plot(-3,3)
```




```
R=E1(-2,1)
```

```
2*R
```

```
(97/4 : -953/8 : 1)
```

```
P=2*R; Q=5*R
```

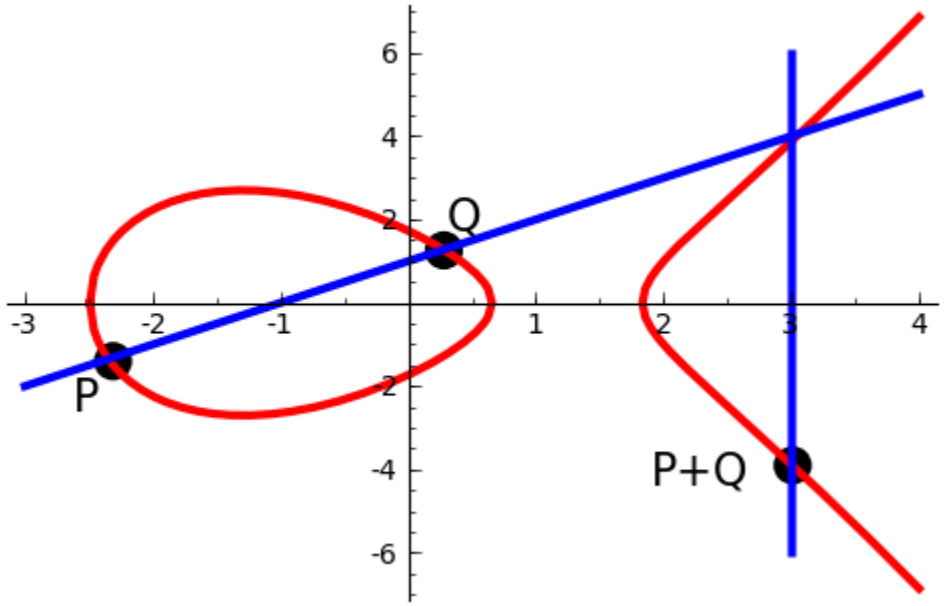
```
P+Q
```

```
(418766370946835830953838/14256497118549422079969 61 :
249255673560374911373576779716250043 9/17022339419912513263590265 4
2156209 : 1)
```

```
E1(-2,-1)+E1(1,1)
```

```
(13/9 : -35/27 : 1)
```

```
G=plot(E0, -3, 4, thickness=3, color='red')
G += line([(-3,-2), (4,5)], rgbcolor='blue', thickness=3)
G += line([(3,-6), (3,6)], rgbcolor='blue', thickness=3)
G += point([(-2.31,-1.4), (0.28,1.26), (3.01,-3.9)],
pointsize=150, rgbcolor='black')
G += text("P", (-2.5,-2.2), rgbcolor='black',fontsize=16)
G += text("Q", (0.45,2.1), rgbcolor='black',fontsize=16)
G += text("P+Q", (2.3,-4), rgbcolor='black',fontsize=16)
G.show()
```




```
# My favoriate elliptic curve:   y*(y+1) = x*(x-1)*(x+2)
E = EllipticCurve([0,1,1,-2,0]); E
```

```
Elliptic Curve defined by y^2 + y = x^3 + x^2 - 2*x over Rational
Field
```

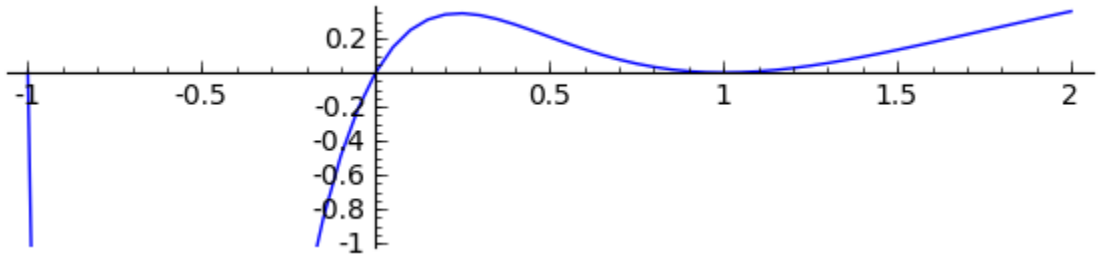
```
L = E.lseries(); L
```

```
Complex L-series of the Elliptic Curve defined by y^2 + y = x^3 +
x^2 - 2*x over Rational Field
```

```
L(2)
```

```
0.360092863578881
```

```
time show(line([(i,L(i)) for i in [-1,-.95,..,2]]), figsize=
[7,1.5], ymin=-1)
```




```
Time: CPU 0.37 s, Wall: 4.37 s
```

```
L.taylor_series(1)
```

```
-1.28158145691931e-23 + (7.26268290635587e-24)*z +
0.759316500288427*z^2 - 0.430302337583362*z^3 -
0.193509313829981*z^4 + 0.459971558373642*z^5 + O(z^6)
```

```
E.rank()
```

```
2
```