Basic solutions

Let us consider a linear program (problem C)

$$\min c[N] \times x[N]$$

subject to conditions:

$$A[M, N] \times x[N] = b[M],$$

$$x[N] \geq 0.$$

A set $N_B \subset N$ is a *basis* if

(i) the columns $A[M, j]$ ($j \in N_B$) are linear independent;

(ii) the columns $A[M, j]$ ($j \bar{\in} N_B$) and vector $b[M]$ are linear combinations of vectors $A[M, j]$ ($j \in N_B$).

If N_B is a basis then the variables x_j for $j \in N_B$ are *basic variables*, x_j for $j \in N_F = N \setminus N_B$ are *non-basic variables*, $A[M, j]$ for $j \in N_B$ are *basic columns*, $A[M, N_B]$ is a *basic matrix*.

A vector $x[N]$ is a *basic solution* if $x_j = 0$ for $j \in N_F$.

1. Let Ω be a closed convex set which does not contain any straight line. Then Ω has an extreme point.

2. If problem C has a solution then it has a solution which is an extreme point of its feasible set Ω.

3. Vector x is a feasible basic solution of problem C iff it is an extreme point of Ω.