The Simplex Method. Improving a basic feasible solution

Let us consider a linear program (problem C)

$$\min c[N] \times x[N]$$

subject to conditions:

$$A[M,N] \times x[N] = b[M],$$

$$x[N] \geq 0.$$

A basis N_B is *not degenerate* if the corresponding basic solution satisfy the following condition: $x_j \neq 0$ for $j \in N_B$.

Assumptions:

(i) Each basis contains m components;

(ii) absence of degeneracy: each feasible basis is not degenerate.

Let N_B be a feasible basis and let x be the corresponding basic solution.

An improving algorithm (one step of the simplex method)

(1) Calculate the vector of the dual variables $y[M]$ from the following system of linear equations:

$$y[M] \times A[M, N_B] = c[N_B].$$

(2) If $y[M] \times A[M,j] \leq c[j]$ for all $j \in N$ then x is an optimal solution of problem (C) and y is an optimal solution of the dual problem.

(3) Find an element to be introduced into the basis: j_0 such that $y[M] \times A[M, j_0] > c[j_0]$.

(4) Find a vector $\lambda[N_B]$ from the following system of linear equations:

$$A[M, N_B] \times \lambda[N_B] = A[M, j_0].$$

(5) Find an element to be removed from the basis: j_1 such that

$$t = \frac{x_{j_1}}{\lambda_{j_1}} = \min_{j \in N_B : \lambda[j] > 0} \frac{x_j}{\lambda_j};$$

If $\lambda_j \leq 0$ for all $j \in N_B$ problem (C) has no solution.

(6) Calculate a new basis $N_B^{'}$ and a new basic solution $x^{'}$:

$$N_B^{'} = N_B \cup j_0 \setminus j_1$$

$$x_{j0}^{'} = t,$$

$$x_j^{'} = x_j - t\lambda_j \ (j \in N_B)$$

In the absence of degeneracy we have:

(i) $cx^{'} < cx$;

(ii) the simplex method leads to an optimal basic feasible solution in a finite number of steps.