Problems

1. Check if the following functions have a saddle point

$$F(x,y) = (x-y)^2, \;\; 0 \le x, y \le 1;$$

$$F(x,y) = (x-y)^2 - 0.5x^2, \;\; -1 \le x \le 1, \; -0.5 \le y \le 0.5;$$

2. Find a solution of the following problems:

(a)

$$\max(3x_1 + 2x_2)$$

subject to

$$(x_1-2)^2 + (x_2-1)^2 \le 9, \; x_1, x_2 \ge 0;$$

(b)

$$\min(8x_1^2 + 2x_2^2)$$

subject to

$$x_1^2 + x_2^2 \le 9, \; 1 \le x_1 \le 2, \; x_2 \ge 1.$$

3. Perform one step of the Newton method for the minimization of the functions:

(a)

$$f(x) = x_1^4 + x_2^2.$$

The initial point is $x_0 = (1,1)$.

(b)

$$f(x) = (x_1^4 + x_2^4 + x_1^2 + 2x_2^2 - x_1x_2 + x_1 + x_2).$$

The initial point is (1,0).

4. Perform one step of the feasible directions method for problems (2a) and (2b) beginning at the point $x_0 = (1,1)$.

5. Find a and b such that the function

$$f(x) = \begin{cases} b(x-\alpha), & x \le \alpha, \\ a(x-\alpha), & x > \alpha \end{cases}$$

is a convex function.

6. Find an area where the following functions are convex

$$f(x) = \sum_{i=1}^{n} x_j \ln x_j;$$

$$f(x, y) = \sin(x) + sin(y);$$

$$f(x, y) = x^2 - 3xy + y^2.$$

$$f(x) = x_1^4 + x_2^4 - x_1 x_2$$

7. Construct a dual problem for the following problem:

$$min \sum_{i=1}^{n} e^{x_i}$$

subject to

$$\sum_{i=1}^{n} x_i \geq 1.$$

8. We have a bar 140 cm, and we can cut it into blanks:
20 cm, price \$3,
40 cm, price \$8,
60 cm, price \$12,
100 cm, price \$16.
Find a cutting of maximal worth.

9. An electronic system consists of 4 components. The components have the following probabilities of failure and prices:
Component 1: Probability 0.2, price \$10;
Component 2: Probability 0.15, price \$10;
Component 3: Probability 0.1, price \$15;
Component 4: Probability 0.05, price \$20.

It is possible to use every component in parallel. We have a budget \$50. What and how many components should be used in parallel to maximize the reliability of the system?

10. Check if point $(\frac{33}{7}, \frac{6}{7})$ is a solution of the problem

$$\min(4(x_1 - 6)^2 + 1.5(x_2 - 2)^2),$$

$$
\begin{aligned}
0.5x_1 + x_2 &\leq 4, \\
3x_1 + x_2 &\leq 15, \\
x_1 + x_2 &\geq 1, \\
x_1 \geq 0,\ x_2 &\geq 0.
\end{aligned}
$$