Convex Sets

A set C is a *convex set* if $\lambda x + (1 - \lambda)y \in C$ for each $x \in C$, $y \in C$ and $0 < \lambda < 1$. A sum $\lambda_1 x_1 + \ldots + \lambda_m x_m$ is called a *convex combination* of $x_1 \ldots x_m$ if the cofficients λ_i are non-negative and $\lambda_1 + \ldots + \lambda_m = 1$.

Properties

1. If A and B are convex sets then the following sets are convex:

$$A \cap B,$$

$$A + B,$$

$$\alpha A,$$

$$\bar{A},$$

$$Int(A).$$

2. A set C is convex if and only if it contains all the convex combinations of its elements.

Cones

A set K is a *cone* if $\lambda x \in K$ for each $x \in K$ and $\lambda > 0$.
A *convex cone* is a cone which is a convex set.

The intersection of an arbitrary collection of convex cones is a convex cone.

Extreme points

Let C be a convex set. A point $x \in C$ is an *extreme point* of C iff there is no way to express x as a convex combination $\lambda y + (1 - \lambda z)$ where $y \neq z$.

Convex hull

The intersection of all the convex sets containing a given subset S is called the *convex hull* of S and is denoted by $Co(S)$.

The convex hull of a finite set is colled a *polytope.*

Properties

1. $Co(S)$ consists of all the convex combinations of the elements of S.

2. (Caratheodory's Theorem) Let S be any set in R^n. Then any point of $Co(S)$ can be represented as a convex combination of $(n+1)$ of the points in S (not necessary distinct).