

TITLES and ABSTRACTS

Marcelo Aguiar, Texas A&M University
Coxeter Hopf monoids

Eli Aljadeff, Technion
Polynomial identities, graded algebras and codimension growth

Abstract:

Given an associative algebra W over a field F of characteristic zero we consider the ideal $\text{id}(W)$ of polynomial identities of W in the free algebra over F (with countably many variables). One of the most important ways to measure the size of $\text{id}(W)$ is by the codimension growth. It measures how big are the spaces of multilinear monomials in n variables modulo identities. So if $\text{id}(W)$ is zero, (i.e. no identities), then the growth is factorial. Otherwise is exponential or polynomial and the exponent is denoted by $\exp(W)$ (bigger is the ideal of identities, lower is the exponent). Now if W is G -graded, where G is any finite group, one may consider the exponents of W and of W_e (the identity component of W). In 2004 Bahturin and Zaicev conjectured that

$$\exp(W) \leq |G|^2 \exp(W_e)$$

The purpose of this lecture is exhibit several results in polynomial identities and graded polynomial identities which are used in the proof of this conjecture.

Nicolas Andruskiewitsch, National University of Cordoba, Argentina
On the classification of finite-dimensional Hopf algebras

Margaret Beattie, Mount Allison University
Twistings of Hopf algebras with a coalgebra projection

Abstract:

Let H be a Hopf algebra over a field K of characteristic 0 and let A be a bialgebra or Hopf algebra such that H is isomorphic to a sub-Hopf algebra of A and there is an H -bilinear coalgebra projection π from A to H which splits the inclusion. Then $A \cong R\#_{\xi}H$ where R is the pre-bialgebra of coinvariants. We study the deformations of A by an H -bilinear cocycle and ask when A can be deformed to a Radford

biproduct. If A has the dual Chevalley property, then we can describe A as a twist of a bosonization of a dual quasi-bialgebra in the category ${}^H_H\mathcal{YD}$ with H , but in general we do not know if A can be deformed to a Radford biproduct.

However, for many pointed Hopf algebras, by the classification results of Andruskiewitsch and Schneider, and by results of Masuoka and Grünenfelder and Mastnak, this will be the case and so we seek instructive examples in this context. We consider liftings of $\mathcal{B}(V)\#K[\Gamma]$ where Γ is a finite abelian group, V is a quantum plane and $\mathcal{B}(V)$ is its Nichols algebra, and explicitly construct the cocycle which twists the Radford biproduct into the lifting.

This is joint work with A. Ardizzoni and C. Menini, U. Ferrara.

Nir Ben-David, Technion

On groups of central type and Involutive Yang–Baxter groups: a cohomological approach

Abstract:

A finite group G is of central type (in the non-classical sense) if it admits a cohomology class $[c] \in H^2(G, C^*)$ (G acts trivially on C^*) such that the twisted group algebra $C^c[G]$ is simple. We call $[c]$ nondegenerate. Groups of central type play a fundamental role in the classification of semisimple triangular complex Hopf algebras and can be determined by their representation-theoretical properties. There is a construction of groups of central type, due to Etingof and Gelaki, which uses Involutive Yang-Baxter (IYB) groups. A finite group Q is IYB group, if Q acts on an abelian group A and there exist a bijective 1-cocycle $\pi : Q \rightarrow \hat{A}$ (in particular $|Q| = |A|$). Etingof and Gelaki showed that the semi direct product of A and Q is a group of central type. We generalize this construction for some non-splitting extensions and strictly extend the above family of groups of central type. Moreover, we give a correspondence between bijective and nondegenerate classes.

Another result is a construction of IYB groups by cohomological methods. More precisely, given an IYB group Q , we construct extensions of abelian groups by Q which are IYB.

Joint work with Yuval Ginosar

*Gabriella Böhm, Research Institute for Particle and Nuclear Physics,
Budapest*

The weak theory of monads

Abstract:

While many familiar constructions in Hopf algebra theory fit the so-called ‘formal theory of monads’ due to Lack and Street, generalizations to weak Hopf algebras are beyond its scope. The aim of this talk is to introduce an extended ‘weak’ theory of monads – providing a categorical framework also for constructions related to weak Hopf algebras.

References.

- [1] G. Bohm. The weak theory of monads. Adv. in Math. in press. arXiv:0902.4192.
 - [2] G. Bohm, S. Lack and R. Street. Weak bimonads. arXiv:1002.4493.
 - [3] G. Bohm, S. Lack and R. Street. The weak world. in preparation.
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Ami Braun, Haifa University

Toral p-Lie algebra polynomial invariants

Abstract:

Let $T < \mathfrak{gl}(V)$, be a Lie algebra torus, where V is a finite dimensional vector space over a field of prime characteristic. The action of T on V extends to an action of T , by derivations, on the symmetric algebra $S(V)$. We shall explore when the fixed ring (or ring of invariants) of this action has the polynomial/complete intersection/Gorenstein properties. This is in analogy to similar questions for finite group actions on $S(V)$. The relevant (now classical) theorems in the complex field case are due to Shephard–Todd–Chevalley, Watanabe, Stanley, Kac, Nakajima and Gordiev.

Stefaan Caenepeel, Vrije University, Brussels

Cohomology with values in restricted Picard groupoids, with applications to Galois coobjects, monoidal categories and Azumaya algebras

Bill Chin, DePaul University

Galois coverings of pointed coalgebras

Abstract:

We introduce the concept of Galois coverings of pointed coalgebras. This is done by considering topological coverings of quivers and then passing to subcoalgebras of the path algebra. The theory shows that Galois coverings of coalgebras can be concretely expressed by smash coproducts using vertex liftings and coactions of automorphism groups of coverings. The theory of Galois coverings is seen to be equivalent to group gradings of coalgebras. The theory gives rise to many gradings, including a universal one, of a given subcoalgebra of a path coalgebra.

Shlomi Gelaki, Technion

Semisimplicity in symmetric tensor categories

Abstract:

Let n be a positive integer, and let k be a field of characteristic 0 or $p > n$. Let G be any group, let $C = \text{Rep}(G)$ be the category of finite dimensional representations of G over k , and let V be an object of C . J.-P. Serre proved that if the n th exterior power of V (respectively, the n th symmetric power of V) is semisimple and the dimension of V is not equal in k to $2, 3, \dots, n$ (respectively, $-n, \dots, -2$), then V is semisimple. Moreover, Serre explained that the same results hold for Lie algebras, and conjectured that they should hold more generally in any symmetric tensor category C over k . Recently we proved a generalization of Serre's conjecture. Namely, we replaced $\text{Rep}(G)$ by any symmetric tensor category C over k , as suggested by Serre, and moreover replaced the n th exterior power and symmetric power by any Schur functor S_λ (here λ is a partition of n). In my talk I will state our results and explain its proofs.

Istvan Heckenberger, University of Marburg

Old and new finite-dimensional Nichols algebras over non-abelian groups

Christian Kassel, CNRS and Université de Strasbourg

Flatness properties of the generic Hopf Galois extension

Abstract:

In a paper published in *Advances in Math.* 208 (2008), 1453–1495, to each Hopf algebra H Eli Aljadeff and I attached a subalgebra B of the free commutative Hopf algebra S generated by the coalgebra underlying H . The commutative algebra B is the subalgebra of coinvariants of a generic Hopf Galois extension. In my talk I'll report on joint work with Akira Masuoka (arXiv:0911.3719), in which we give conditions under which S is faithfully flat, or even free, as a B -module. Under these conditions, B is a versal deformation space for all forms of H .

Vladislav Kharchenko, National University of Mexico

Coideal subalgebras in quantum groups

Leonid Krop

On the trace of the antipode

For a simply linked datum \mathcal{D} of nilpotent type let $H = H(\mathcal{D})$ be the associated Hopf algebra. We derive a formula for the trace of the antipode for every simple (self-dual) H -module. In the case of a datum on a two-dimensional space our results give an explicit formula for the second indicator defined in a recent paper of Y. Kashina, S. Montgomery and Siu-Hung NG. The talk is based in part on joint work with A. Jedwab

Burkhard Külshammer, University of Jena, Germany

The depth of subgroups and subalgebras

Abstract: A ring extension $A \subseteq B$ is said to have depth one if B is isomorphic to a direct summand of A^n as an A - A -bimodule, for some positive integer n . In my talk, I plan to present group-theoretic characterizations of this property in the case $kH \subseteq kG$ where H is a subgroup of a finite group G and k is a field.

A ring extension $A \subseteq B$ is said to have depth two if $B \otimes_A B$ is isomorphic to a direct summand of B^n for some positive integer n , both as an A - B -bimodule and a B - A -bimodule. It is easy to see that depth one implies depth two. I plan to show that the ring extension $kH \subseteq kG$

has depth two if and only if H is a normal subgroup of G . This fact can be generalized to Hopf algebras.

I will also introduce the notion of higher depth. For a subgroup H of a finite group G , the depth of $kH \subseteq kG$ is always finite. I will present various characterizations of the depth in the case of group algebras. My talk will be a report on joint work with R. Boltje (Santa Cruz), S. Burciu (Bucharest), S. Danz (Oxford) and L. Kadison (San Diego).

Martin Lorenz, Temple University

Ring theoretic methods in the representation theory of Hopf algebras

Avinoam Mann, The Hebrew University of Jerusalem

Curiouser and curiouser

Partition formulae connected with non-abelian groups

Abstract:

In 1938 P. Hall published the following remarkable identity, which he described as “rather curious”. Fix a prime p , and let G vary over all finite abelian p -groups. Then

$$\sum_G \frac{1}{|G|} = \sum_G \frac{1}{|\text{Aut}(G)|}.$$

Since the number of abelian groups of order p^n is $\pi(n)$, the number of partitions of n , another way of writing the above formula is

$$\sum_G \frac{1}{|\text{Aut}(G)|} = \sum \frac{\pi(n)}{p^n}.$$

We give analogous formulae for some families of non-abelian p -groups.

Akira Masuoka, University of Tsukuba

On affine supergroups

Abstract:

Let us work over an arbitrary field k of characteristic $\neq 2$. An *affine supergroup* is a representable functor, say G , from the category of super-commutative superalgebras (over k) to the category of groups. It is uniquely represented by a super-commutative Hopf superalgebra, say A . This A has the largest quotient ordinary commutative Hopf algebra, \overline{A} . Let W^A denote the odd part of the cotangent super-vector

space of G at 1. In my paper published 2005, I proved that there is a counit-preserving isomorphism $A \simeq \overline{A} \otimes \wedge W^A$ of left \overline{A} -comodule superalgebras. I will discuss some new applications of this isomorphism. Part of the results is from a joint work with Alexander Zubkov.

Claudia Mennini, University of Ferrara

Hopf algebras with the dual Chevalley property are gauge deformations

Abstract:

Let K be a field. It is well known that, given a gauge transformation $\zeta : A \otimes A \rightarrow K$, one can twist the multiplicative structure of a bialgebra A by ζ to obtain A^ζ which is not a bialgebra in general, but a dual quasi-bialgebra.

Let A be a Hopf algebra and let H be its coradical. Assume that H is a sub-Hopf algebra of A (i.e. A has the dual Chevalley property) and that H is also semisimple.

Then we produce a gauge transformation ζ such that

$$A^\zeta \cong Q \# H$$

i.e. A^ζ is obtained as a Radford–Majid bosonization like procedure by H of a connected dual quasi-bialgebra Q in the category of left-left Yetter–Drinfeld modules over H .

Thus we get $A \cong (Q \# H)^\gamma$, where $\gamma = \zeta^{-1}$ is the convolution inverse of ζ .

This is part of a joint research with A. Ardizzoni and M. Beattie.

Walter J. Michaelis, University of New Orleans

Commuting exponentials in a Lie group

Abstract:

This is a joint work with Karl H. Hofmann

Two commuting real matrices A and B have commuting exponentials $\exp A$ and $\exp B$, a fact observed for instance in linear algebra or differential equations courses. The converse implication is false. A clarification of this phenomenon is proposed that makes use of the theory of the exponential function $\exp : \mathfrak{g} \rightarrow G$ of a real Lie group G and its singularities. In Section 1, a catalog of low-dimensional examples illustrates various ways that, for two elements $X, Y \in \mathfrak{g}$, the commuting of $\exp X$ and $\exp Y$ in G may fail to entail the commuting of X and Y in

\mathfrak{g} . In Section 2, consequences of the relation $[\exp X, \exp Y] = \mathbf{1}$ are inspected, whereby certain regularity assumptions on X and Y are made. A regular element Y of the Lie algebra \mathfrak{g} determines a Cartan subalgebra $\mathfrak{h} = \mathfrak{g}^0(Y)$ of \mathfrak{g} and a certain subgroup \mathcal{W}_Y of the (finite!) Weyl group of \mathfrak{g} with respect to the Cartan subalgebra \mathfrak{h} . If, additionally, the exponential function is regular at X and at Y , then the ordered pair (X, Y) is said to be in *general position*. If (X, Y) is in general position, then the relation $[\exp X, \exp Y] = \mathbf{1}$ in G permits the definition of a certain element $w(X, Y) \in \mathcal{W}_Y$. Let $\mathfrak{z}(\mathfrak{g})$ denote the center of \mathfrak{g} . It is shown that, if $\exp X$ and $\exp Y$ commute in G for (X, Y) in general position, then $[X, Y] \in \mathfrak{z}(\mathfrak{g}) \cap [\mathfrak{h}, \mathfrak{h}]$ iff $w(X, Y) = \mathbf{1}$. Write $H =: \exp \mathfrak{h}$, and let $Z(G)$ denote the center of G . If the identity component of $Z(G) \cap [H, H]$ is simply connected, and if $\exp X$ and $\exp Y$ commute for (X, Y) in general position, then $[X, Y] = 0$ iff $w(X, Y) = \mathbf{1}$. If G is simply connected compact, then $[\exp X, \exp Y] = \mathbf{1}$ and $[X, Y] = 0$ are equivalent for all pairs (X, Y) in general position. In $\mathrm{SO}(3)$ this is not the case; here $|\mathcal{W}_Y| = 2$. In Section 3, examples show that the validity of the equation $\exp X \exp Y = \exp(X + Y)$ has no implications whatsoever in the direction of the commuting of $\exp X$ and $\exp Y$. Finally, in Section 4, it is shown that, for a simply connected Lie group G , the commuting of $X, Y \in \mathfrak{g}$ and that of $\exp X, \exp Y \in G$ are equivalent properties for *all* X and Y if and only if the exponential function is injective. This class of Lie groups was characterized in terms of other properties by Dixmier and by Saito, independently, in 1957.

Susan Montgomery, University of Southern California

Frobenius-Schur indicators and the trace of the antipode

Abstract:

Much of the early work on (finite-dimensional) Hopf algebras was concerned with the possible values of $\mathrm{Tr}(S^2)$, as a step to the Larson-Radford theorem on semisimplicity and $S^2 = \mathrm{id}$. We propose here that the values of $\mathrm{Tr}(S)$ itself are important even in the case of a semisimple Hopf algebra H .

First, $\mathrm{Tr}(S)$ appears in the Frobenius-Schur indicator $\nu(V)$ of an irreducible representation V of H . If S_V is the map induced by S on $\mathrm{End}(V)$, then $\mathrm{Tr}(S_V) = \nu(V) \dim(V)$, and so $\mathrm{Tr}(S) = \sum_V \nu(\chi) \dim(V)$ [LM]. Note that a Hopf algebra is “totally orthogonal” (all indicators $+1$) if and only if $\mathrm{Tr}(S_V) = \sum_V \dim(V)$. Clearly for $H = kG$, $\mathrm{Tr}(S) = i_G > 0$, where i_G is the number of x in G such that $X^2 = 1$.

In work with Jedwab, we showed that for H a bismash product constructed from the factorizable group $L = FG$, in fact $\text{Tr}(S_H) = i_L > 0$. In joint work with Kashina and Ng, we raise the question of when $\text{Tr}(S)$ is positive. We show that $\text{Tr}(S)$ can be negative for a bicrossed product, although we do not know if $\text{Tr}(S)$ can be 0. We prove that if the representation category of H is modular, then $\text{Tr}(S) \geq 0$, and if $H = D(A)$ then $\text{Tr}(S_H) = \text{Tr}(S_A)^2$. Thus having non-negative trace is a property of well-behaved Hopf algebras.

We apply some of these results to prove a Hopf analog of a main preliminary step in the Brauer-Fowler theorem for finite groups. That is, if $\dim(H)$ is even, we prove there exists some self-dual irreducible representation V of H such that

$$\dim(V) \leq (\dim(H) - 1) |\text{Tr}(S) - 1|^{-1}.$$

Sonia Natale, National University of Cordoba, Argentina

Hopf algebra deformations of binary polyhedral groups

Abstract:

This is joint work with J. Bichon. We show that semisimple Hopf algebras having a self-dual faithful irreducible comodule of dimension 2 are always obtained as abelian extensions with quotient \mathbb{Z}_2 . We prove that nontrivial Hopf algebras arising in this way can be regarded as deformations of binary polyhedral groups and describe its category of representations. We also prove a strengthening of a result of Nichols and Richmond on cosemisimple Hopf algebras with a 2-dimensional irreducible comodule in the finite dimensional context. Finally, we give some applications to the classification of certain classes of semisimple Hopf algebras.

Siu-Hung Ng, Iowa State University

On gauge invariants of finite-dimensional Hopf algebras

Abstract:

Frobenius–Schur indicators can be defined for each object in a pivotal category, and they are invariants of pivotal categories. However, it remains unclear how this notion can be extended to finite tensor categories. In this talk, we introduce some invariants for the tensor categories of representations of finite-dimensional Hopf algebras. These invariants coincide with the higher Frobenius–Schur indicators of the regular representation in the case of complex semisimple Hopf algebras.

We will demonstrate some properties of these invariants and their applications.

Part of this talk is based on joint work with Y. Kashina and S. Montgomery.

Uri Onn, Ben-Gurion University of the Negev
 p -adic, real and quantum Grassmannians

Abstract:

Let F be a local field. The action of $\mathrm{GL}(n, F)$ on the Grassmann variety $\mathrm{Gr}(m, n, F)$ induces a continuous representation of the maximal compact subgroup of $\mathrm{GL}(n, F)$ on the space of L^2 -functions on $\mathrm{Gr}(m, n, F)$. The irreducible constituents of this representation are parameterized by the same underlying set both for archimedean and non-archimedean fields. In a quite mysterious way the archimedean and non-archimedean theories are interpolated by quantum Grassmannians. In this talk I shall describe this phenomena.

Donald S. Passman, University of Wisconsin-Madison
The Semiprimitivity Problem for Group Algebras

Abstract:

The semiprimitivity problem for group algebras $K[G]$ of infinite groups has been studied for over fifty years. In this talk, I will give a history of the problem, discussing what is known and what might be next.

David Radford, University of Illinois at Chicago
Some generalizations of quasitriangular and co-quasitriangular Hopf algebras and where the theory of their resulting invariants of knots and links stands at this point

Abstract:

We review the generalizations introduced by Kauffman, or Kaffman and Radford, which account for invariants of 1-1 tangles, knots, or links. These generalizations are certain algebras or coalgebras. Of particular interest is how they, together with additional structure for computing invariants, relate to matrix algebras, Drinfel'd doubles, or their dual coalgebras. One is led to consider invariants of knots and links which lie in a vector space.

Serban Raianu, California State University, Dominguez Hills
Integrals and the Antipode

Abstract:

Questions of Moss Sweedler on the uniqueness of integrals and the bijectivity of the antipode were answered, in this order, by John Sullivan and David Radford. Radford's proof for the bijectivity of the antipode used Sullivan's result on the uniqueness of integrals. Proving the bijectivity of the antipode without using the uniqueness of integrals enables a proof for the uniqueness that is similar to the classical proof in the case of Haar measures on a compact group. The talk is partially based on joint work with Miodrag Iovanov.

Louis Rowen, Bar-Ilan University
Full quivers of representations of algebras

This is joint work with Alexei Belov and Uzi Vishne.

Hans-Juergen Schneider, University of Munich
Root systems and Weyl groupoids of Nichols algebras

Abstract:

This is a report on joint work with N. Andruskiewitsch and I. Heckenberger, and with I. Heckenberger. A fundamental problem in Hopf algebras is the investigation of the Nichols algebra of a direct sum of finitely many irreducible Yetter–Drinfeld modules over a Hopf algebra with bijective antipode, in particular over the group algebra of a finite or an abelian group. The main technical tools are the associated root system and Weyl groupoid first introduced by I. Heckenberger for diagonal braidings. I will explain applications to the classification of Hopf algebras. In a joint paper with I. Heckenberger we explicitly describe right coideal subalgebras of the Nichols algebra in terms of the Weyl groupoid, establish a PBW type basis in our general context, and answer a question of Kharchenko on quantum groups. We do not rely on case by case considerations. Hence we present a new approach also to the special case of the plus part of the deformed enveloping algebra of a semisimple Lie algebra.

Zhu Shenglin, Fudan University, Shanghai

On Smash Products of Transitive Module Algebras

Abstract:

Let H be a semisimple Hopf algebra over a field of characteristic 0, and A a transitive H -module algebra. In the case that A has 1-dimensional ideal, we prove that the smash product $A\#H$ is a full matrix algebra over some right coideal subalgebra N of H . The correspondences between A and such N , and the special case when $A = k(X)$ of function algebras over a set X are considered.

Lance Small, University of California, San-Diego

Affine algebras and infinite dimensional division algebras

Abstract:

We will survey recent developments in the study of affine algebras with low GK dimension like Bell's proof of the dichotomy conjecture: affine, prime, noetherian rings of GK 2 are primitive or PI. Some of the methods used in study of low GK dimension affine are useful in the study of infinite dimensional division rings. We will report on this, too.

Daniel Sternheimer, Keio University and University of Bourgogne

From Lie groups to Hopf algebras, their deformations and physical applications

Earl J. Taft, Rutgers University

Recursive Sequences and Hopf Algebras

Abstract:

We will survey the interactions of recursive sequences and Hopf-like structures. This includes: linearly recursive sequences as Hopf algebra (divided-power product) and as bialgebra (pointwise product); polynomially recursive sequences as topological bialgebra; linearly recursive sequences as Lie coalgebra and Lie bialgebra dual to the Witt algebra; the sequence of Frobenius-Schur indicators of a finite-dimensional Hopf algebra; linearly recursive sequences in several variables; quantum convolution of linearly recursive sequences in one and two variables.

Sara Westreich, Bar-Ilan University

Structure constants related to symmetric Hopf algebras

Abstract:

(Joint work with M. Cohen)

Let k be an algebraically closed field of characteristic 0. We study the structure constants for semisimple quasitriangular Hopf algebras over k and generalize them to non-semisimple factorizable ribbon Hopf algebras. This is done from the point of view of symmetric algebras, such as group algebras. In particular we consider general fusion rules which are structure constants associated to products of irreducible characters and structure constants associated to generalizations of class sums and conjugacy classes. Our methods are reminiscent on one hand of the methods employed when analyzing tilting modules for certain quantum groups and of “splitting modules” for certain Drinfeld doubles on the other.

The family of irreducible characters is divided in two according to the vanishing of their quantum dimension. The fusion rules on the character ring are computed with respect to this division.

Yinhuo Zhang, Hasselt University

Derived H -module endomorphism rings

Abstract:

(joint with Jiwei He and F. Van Oystaeyen)

In this talk we discuss when a homological property is preserved under Hopf Galois extensions. We show that the Kozul and Gorenstein properties are preserved under Hopf Galois extension when the Hopf algebra is semisimple.
