ACTION OF TRANSLATION FUNCTORS ON SIMPLE MODULES FOR THE PERIPLECTIC SUPERALGEBRA $\mathfrak{p}(n)$

Action of translation functors on simple modules

In this note we consider the finite-dimensional integrable representations of the periplectic superalgebra $\mathfrak{p}(n)$ and describe the action of translation functors Θ_k and their composites on the simple $\mathfrak{p}(n)$-modules $L_n(\lambda)$ (notation as in [BDE+16, ES19]).

In the tables below we list the weight diagrams for the pairs $(\lambda, \mu) \in \Lambda_n \times \Lambda_n$ for which

$$[\widetilde{\Theta} L_n(\lambda) : L_n(\mu)] \neq 0$$

for the translation functors $\widetilde{\Theta} := \Theta_i,\ \Theta_{i+1}\Theta_i,\ \Theta_{i-1}\Theta_i$. In each case, we have:

$$[\widetilde{\Theta} L_n(\lambda) : L_n(\mu)] = 1$$

(up to change of parity), as shown in [BDE+16].

The weight diagrams below should be read as follows. For any weight diagram d_λ with positions $i-2, i-1, \ldots$ as in the left column, we list in the central column all weight diagrams d_μ such that

$$\exists z \in \mathbb{Z}/2\mathbb{Z} :\ [\widetilde{\Theta} L_n(\lambda) : \Pi^z L_n(\mu)] \neq 0$$

(note that only one such z exists, by the results of [BDE+16]). When possible, we also list the relevant parity z, based on [BDE+16, Propositions 5.2.1, 5.2.2].

In the weight diagram of d_μ we also draw the relevant shortest solid and dashed arrows as defined in [BDE+16]. In each case, the j-th position differs in d_μ and in d_λ. The positions not marked in neither d_μ nor d_λ coincide.

References

[BDE+16] M. Balagović, Z. Daugherty, I. Entova-Aizenbud, I. Halacheva, J. Hennig, M. S. Im, G. Letzter, E. Norton, V. Serganova and C. Stroppel, *Translation functors and decomposition numbers for the periplectic Lie superalgebra* $\mathfrak{p}(n)$, to appear in Math. Res. Lett.; arXiv:1610.08470.

[ES19] I. Entova-Aizenbud, V. Serganova, *Duflo-Serganova functor and superdimension formula for the periplectic Lie superalgebra*, arXiv:1910.02294 (2019).

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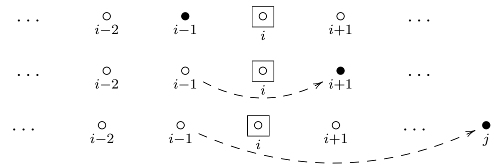
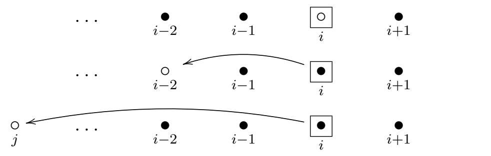
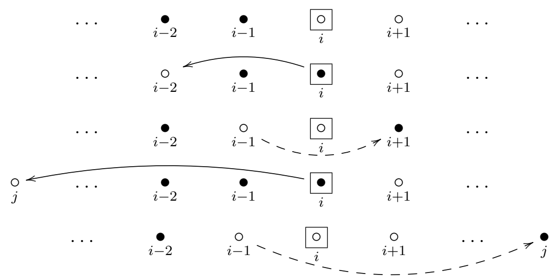
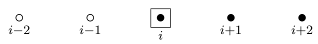
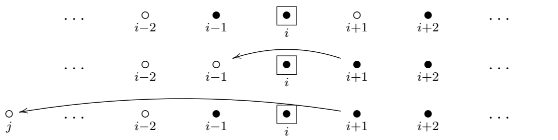
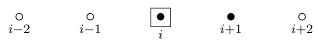
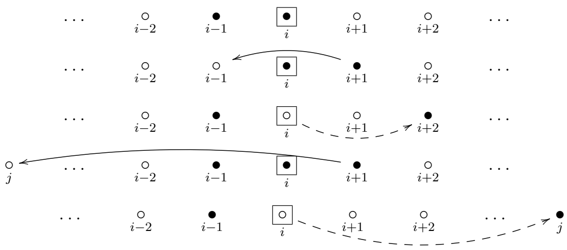
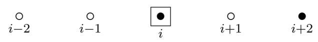
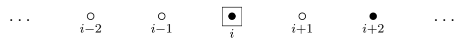
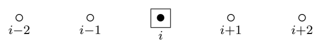
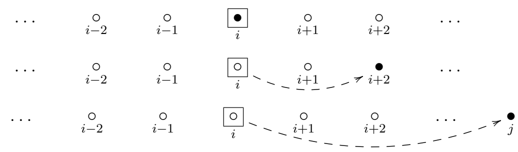
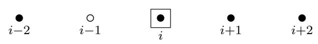
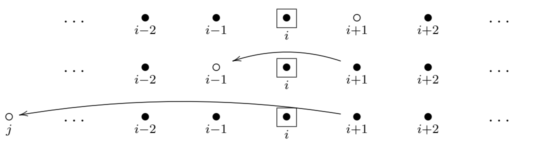
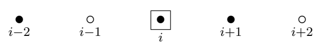
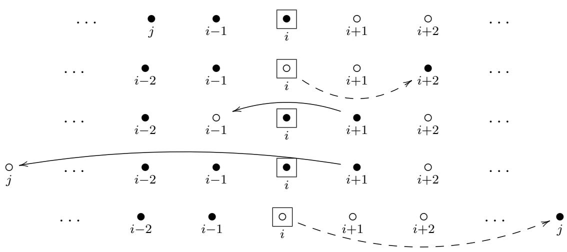
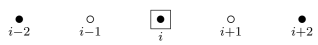
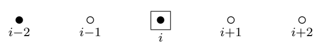
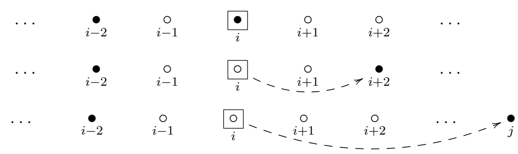
Action of Θ_i

Pairs $(\lambda, \mu) \in \Lambda_n \times \Lambda_n$ for which $[\Theta_i L_n(\lambda) : L_n(\mu)] \neq 0$.

Diagram of d_λ	Diagram of d_μ	Parity z
		$z \equiv i+1 \mod 2$
		$z \equiv i+1 \mod 2$ $z \equiv i \mod 2$ z depends on j
		$z \equiv i+1 \mod 2$ $z \equiv i \mod 2$ z depends on j
		$z \equiv i+1 \mod 2$ $z \equiv i \mod 2$ $z \equiv i \mod 2$ z depends on j z depends on j

Action of $\Theta_{i+1}\Theta_i$

Pairs $(\lambda, \mu) \in \Lambda_n \times \Lambda_n$ for which $[\Theta_{i+1}\Theta_i L_n(\lambda) : L_n(\mu)] \neq 0$.

Diagram of d_λ	Diagram of d_μ	Parity z
		$z \equiv 1 \mod 2$ $z \equiv 0 \mod 2$ z depends on j
		$z \equiv 1 \mod 2$ $z \equiv 0 \mod 2$ $z \equiv 0 \mod 2$ z depends on j z depends on j
		$z \equiv 0 \mod 2$
		$z \equiv 0 \mod 2$ $z \equiv 1 \mod 2$ z depends on j
		$z \equiv 1 \mod 2$ $z \equiv 0 \mod 2$ z depends on j
		$z \equiv 1 \mod 2$ $z \equiv 0 \mod 2$ $z \equiv 0 \mod 2$ z depends on j z depends on j
		$z \equiv 0 \mod 2$
		$z \equiv 0 \mod 2$ $z \equiv 1 \mod 2$ z depends on j