Ergodic Theorems, Group Actions and Applications
EILAT, May 15-20, 2011
Abstracts

Rational weak mixing in infinite measure spaces
Jon Aaronson

Motivated by an example of E. Hopf (in his 1937 book), K. Krickeberg (1967) introduced a topological ratio mixing property which is enjoyed by Markov shifts with the strong ratio limit property.

The theory of weakly wandering sets as developed by Hajian and Kakinotani (1964) shows that there is no measure theoretic version of this ratio mixing property.

Nevertheless, we’ll see that Hopf’s example is ”rationally weak mixing” in a sense which implies that for every pair of measurable sets in a hereditary ring, ratio mixing is satisfied along a subsequence of density one and maybe discuss some questions arising for Markov shifts and other infinite measure preserving transformations.

Abelian and Tauberian Theorems for Cauchy Problems
Wolfgang Arendt

We will give a survey on some results on the asymptotic behavior of semigroups and Cauchy problems. The Tauberian character comes from the fact that the resolvent of the operator is the Laplace transform of the semigroup, or more generally the solution of a Cauchy problem. Such Tauberian theorems attempt to decide from spectral conditions (i.e. conditions on the resolvent) the behaviour of the solution. Spectral conditions, such as countable spectrum gives information on long time behaviour. We will give special attention to positive semigroups where some special results are possible (Perron Frobenius Theory). Not only semigroups, but also resolvent positive with possibly non dense domain will be considered. They are useful to solve the Dirichlet problem on a parabolic cylinder and give information on the asymptotic behavior of the solutions for large time in terms of the given data.
The horocycle flow and counting problems for billiards in genus two
Matt Bainbridge

A basic problem in the study of billiards in rational polygons is the classification of invariant measures for the horocycle flow on the moduli space of genus g Riemann surfaces equipped with a holomorphic one-form. In this talk, I will discuss the classification of such measures which lie on certain special submanifolds of these moduli spaces (called eigenform loci). As an application, we will obtain asymptotics for counting closed billiard paths in certain explicit polygons. This is joint work with Barak Weiss and John Smillie.

Weak mixing of induced interval exchange transformations
Michael Boshernitzan

Maximizing points and coboundaries for an irrational rotation on the Circle
Zoltán Buczolich

Consider an irrational rotation of the unit circle and a real continuous function. A point is declared maximizing if the growth of the ergodic sums at this point is maximal up to an additive constant. In case of two-sided ergodic sums the existence of a maximizing point for a continuous function implies that it is the coboundary of a continuous function. In contrast, we build for the usual one-sided ergodic sums examples in Hölder or smooth classes indicating that all kinds of behaviour of the function with respect to the dynamical system are possible. We also show that generic continuous functions are without maximizing points, not only for rotations, but for the transformation $2x \mod 1$ as well. For this latter transformation it is known that any Hölder continuous function has a maximizing point. This is a joint work with Julien Brémont motivated by a question of Jean-Pierre Conze.
Martingale approximation in $L^p$-Applications

Christophe Cuny

We obtain sufficient conditions under which the partial sums of a stationary process admits an approximation, in $L^p$, $p > 1$, by a martingale with stationary increments. In particular, we investigate conditions similar to the Maxwell-Woodroofe condition. Then, we apply those results, combined with ergodic theorems with rates, in order to obtain generalization of the Marcinkiewicz-Zygmund theorem and some almost sure strong invariance principles.

Ergodic properties of infinite measure-preserving extensions of area-preserving flows

Krzysztof Frączek

I will present some methods for proving ergodicity of skew product extensions of interval exchange transformations of periodic type. I will also deal with a class of smooth flows on non-compact manifolds which are extensions of so called multivalued Hamiltonian flows on compact surfaces of higher genus. These flows have Poincare’ sections for which the first recurrence map is isomorphic to a skew product of an IET and a BV cocycle or a cocycle with logarithmic singularities. This allows us to prove a sufficient condition for ergodicity whenever the IET has periodic type. My talk will be based on two papers joint with J.-P. Conze and C. Ulcigrai.

Structure of Markov operators with applications to combinatorics and fractals

Hillel Fürstenberg

Group boundaries, revisited

Alex Furman

The notion of a boundary of a group plays an important role in the study of large groups, their lattices, and their actions. In particular, $G$-boundaries appear in investigations of random walks, in proofs of super-rigidity results, in computations of bounded cohomology etc. While examples of boundaries invariably include semi-simple Lie group $G$ acting on $B = G/P$, there exists a number of different ways to formalize the properties that make a $G$-space into a usable $G$-boundary for a general $G$.

In a joint work with Uri Bader a new, or revised, boundary formalism is proposed (involving a notion of Ergodicity with Separable Metric Coefficients), and used for superrigidity results and for problems of Lyapunov spectrum.

On Hilbert dynamical systems

Eli Glasner

I will present a joint work with Benjy Weiss where we analyze Hilbert dynamical systems. In particular two corollaries of this analysis will ensue: Returning to a classical question in Harmonic Analysis we strengthen an old result of Walter Rudin. We show that there exists a weakly almost periodic function on the group of integers $\mathbb{Z}$ which is not in the norm-closure of the algebra $B(\mathbb{Z})$ of Fourier-Stieltjes transforms of measures on the dual group $\hat{\mathbb{Z}} = \mathbb{T}$, and which is recurrent. We also show that there is a Polish monothetic group which is reexively but not Hilbert representable.
Endomorphisms, Invariant Splittings and the Central Limit Theorem

Mikhail Gordin

Let we are given a measure preserving transformation of a probability space which for simplicity we will assume to be exact. We are interested in conditions on a square integrable function on this probability space under which the central limit theorem holds. It is known that sometimes a sufficient condition for the central limit theorem to hold can be made more precise in presence of a dynamically invariant splitting (that is an orthogonal decomposition of the space of square integrable functions which is stable under the isometry defined by the endomorphism and the adjoint coisometry). Note that only infinite splittings are of interest in this respect and that a finer splitting should give more than a coarser ones. The existence of some finest splitting is a trivial consequence of the spectral structure of such an isometry: such a splitting is given by any orthogonal decomposition of the wandering subspace into one-dimensional subspaces. We are going to discuss in this context less arbitrary splittigs which can be constructed against a choice of several operators subject to some relations. Together with the dynamical isometry they generate one of Cuntz’s C*-algebras and lead to an invariant splitting. In some classical examples (endomorphisms of compact commutative groups) under appropriate choice of auxillary operators, this reduces to known decompositions in terms of Fourier analysis.

Recurrence and ergodicity of random walks on linear groups

Yves Guivarc’h

We consider a random walk $W$ on a locally compact group $G$ and we recall the known results on the recurrence of $W$ and polynomial growth of $G$. Using the projection of $W$ on $G/H$, where $H$ is a closed subgroup, we describe some new results if $G$ is a subgroup of a linear group over a local field. We consider also the recurrence properties of random walks on homogeneous spaces, and we describe the situation for a few examples.
Rates of decay in Mean Ergodic Theorem
Markus Haase

We will present a general framework for the study of rates of decay in mean ergodic theorems for discrete and strongly continuous operator semigroups. In particular, we will show how to unify and generalize results due to Assani, Cohen, Cuny, Derriennic, and Lin dealing with rates in mean ergodic theorems in a number of cases.

This is joint work with A. Gomilko and Y. Tomilov.

Random graphs, equivalence relations and the Liouville property
Vadim Kaimanovich

The theory of measured discrete equivalence relations provides a natural setup for studying random graphs. Although the foundations of this theory (and its continuous counterpart which deals with measured foliations) were laid out much earlier, it is only now that probabilists are becoming aware of advantages of this approach. I will talk about several recent results on graphed equivalence relations with the Liouville property (absence of leafwise non-constant bounded harmonic functions). More precisely, in this situation (1) the rate of escape is completely described by a certain cocycle of the equivalence relation; (2) the corresponding measure on the space of random rooted graphs can be approximated by an appropriate sequence of finite graphs.

Nonintersecting splitting sigma-algebras in a non-bernoulli transformation
Steven Kalikow

Given a measure preserving transformation $T$ on a Lebesgue sigma algebra, a complete $T$ invariant sub sigma algebra is said to split if there is another complete $T$ invariant sub sigma algebra on which $T$ is Bernoulli which is completely independent of the given sub sigma algebra and such that the two sub sigma algebras together generate the entire sigma algebra. It is easily shown that two splitting sub sigma algebras with nothing in common imply $T$ to be $K$. In 1975 Thouvenot asked if it had to be Bernoulli. The answer is no, it can be the $T$ $T$ inverse transformation.
Characterizations of substitution minimal sets
up to topological conjugacy

Mike Keane

We establish necessary and sufficient conditions for a dynamical system
to be topologically conjugate to either the Morse or the Toeplitz minimal set
(under the shift transformation), and discuss possible extensions for other
substitution systems. This is joint ongoing work with Ethan Coven and
Michelle Lemasurier, some of which has appeared and some which is in prepa-
ration.

Compression of quasianalytic spectral sets of cyclic contractions

Laszlo Kerchy

We continue the study of the class $L_0(H)$ of cyclic quasianalytic con-
tractions, initiated in our paper ”Quasianalytic contractions and function
algebras” to appear in Indiana Univ. Math. J.. We investigate the question,
whether the commutant of every operator $T \in L_0(H)$ contains an operator
$T_0 \in L_0(H)$ such that the quasianalytic spectral set $\pi(T_0)$ includes an arc,
reducing it to a problem in harmonic analysis. This question is strongly
related to the hyperinvariant subspace problem for non-stable contractions.

On the contributions of Georg Bohlmann to the foundations of
probability theory

Ulrich Krengel

The name of Georg Bohlmann (1869-1928) is known to few probabilists.
Yet, It appears that he was the first to give the formal definition of inde-
pendence of n events. He took significant and original steps towards Kol-
mogorov’s axioms. Moreover, Bohlmann gave the now universally accepted
definition of conditional probabilities. Another very interesting contribution
of Bohlmann is a method of smoothing time series, presumably the first
method making use of a penalty term. It resembles the Hodrick-Prescott-
filter. The rediscovery of this paper leads to interesting new questions.
The quenched central limit theorem
for some independent products of transformations
Stephane Le Borgne

Let $G$ be a discrete group acting on a probability space $(X, m)$ and $\mu$ a
probability measure on $G$. We consider the random walk on $X$ defined by $\mu$ in
the following way: the position at time $n$ starting from $x$ is given by $g_n\ldots g_1.x$
and the elements $g_k$ of $G$ are independent with probability distribution $\mu$.
This can also be viewed as random products of transformations of $X$.
In some cases it has been shown that the associated Markov operator

$$P_\mu f(x) = \sum_g f(g.x)\mu(g)$$

has a spectral gap. We show that with such an assumption the central limit
theorem with respect to $m$ holds for almost every sequence $(g_k)_{k\geq 1}$.
Let $P$ be the product measure $\mu^{\otimes N^*}$. If the norm of $P_\mu \otimes P_\mu$ on $L_0^2(X \times X, m \otimes m)$ is strictly less than 1, then, for every function $\varphi$ in $L_0^\infty(m)$, there exists
$\sigma \geq 0$ such that, for $P$ almost every sequence $(g_k)_{k\geq 1}$, we have

$$m\{x \in X / \sum_{k=1}^n \varphi(g_k \ldots g_1.x) \leq \sqrt{nt}\} \rightarrow \int_{-\infty}^t \exp\left(-s^2/2\sigma^2\right) \frac{ds}{\sqrt{2\pi}\sigma}.$$ 

This is a joint work with Jean-Pierre Conze.

Rigidity along sequences
Mariusz Lemanczyk

I will speak about rigidity along sequences in the theory of dynamical
systems. The lecture will be concentrated on the problem which sequences
of natural numbers can or cannot be rigidity sequences for some weakly
mixing transformations. This is a joint work with V. Bergelson, A. del Junco
and J. Rosenblatt.
A universal scheme for pointwise convergence of multiple averages

Emmanuel Lesigne

We give a (random) construction of strictly increasing sequences of integers \((a_n), (b_n)\) such that multiple ergodic averages of the form

\[
\frac{1}{N} \sum_{n=1}^{N} f(T^{a_n}x)g(S^{b_n}x)
\]

do converge pointwise, for any choice of measure preserving commuting transformations \(T\) and \(S\). As far as we know, it is the first example of this type. This is part of a joint work with Nikos Frantzikinakis and Mate Wierdl.

Isometric embeddings \(\ell_2^n\) into \(\ell_p^n\) via group actions

Yuri Lyubich

This is the joint work with Oksana Shatalova.

We consider the Banach spaces over a field \(K\) which is actually \(\mathbb{R}\) or \(\mathbb{C}\), or the noncommutative quaternion field \(\mathbb{H}\). An isometric embedding \(\ell_2^n \to \ell_p^n\) over \(K\) exists if and (for \(m \geq 2\)) only if \(p\) is an even integer and \(n\) is equal to or greater than a quantity \(N_{K}(m, p)\), see [LS1] and the references there. This quantity is still unknown, except for

\[ N_{\mathbb{R}}(2, p) = \frac{p}{2} + 1 \quad ([LV, \mathbb{R}]) \quad (1) \]

and for some concrete values of \(m\) and \(p\). For example,

\[
\begin{align*}
N_{\mathbb{R}}(3, 4) &= 6 \quad ([LV, \mathbb{R}]), \\
N_{\mathbb{C}}(2, 4) &= 4 \quad ([K]), \\
N_{\mathbb{H}}(2, 6) &= 10
\end{align*}
\]

(2)

These and many other examples are due to some nice configurations consisting of \(n\) points on the unit sphere \(S\) of the Euclidean space \(\ell_2^n\). For instance, the formula (1) corresponds to the system of vertices of the regular \(p + 2\)-gon. The first of equations (2) corresponds to the system of vertices of the regular icosahedron, etc.

The following theorem is a general source of the results of such a kind.
**Theorem.** Let $\Gamma$ be a finite group of isometries of the space $\ell_{2,K}^m$, and let $-\text{id} \in \Gamma$. Assume that a point $x \in S$ is a common zero of all $\Gamma$-invariant harmonic polynomial functions of degrees $k \leq p$. Then there exists an isometric embedding $\ell_{2,K}^m \rightarrow \ell_{p,K}^n$ with $n = \frac{1}{2} |\Gamma x|$.

An important application of this theorem is the existence of an isometric embedding $\ell_2^2 \rightarrow \ell_{12}^{10}$. In an equivalent form this is an affirmative answer to a question from algebraic combinatorics [BH]. In [LS2] an orbit of the binary icosahedral subgroup of $SU(2)$ was used to this end. It turns out that this embedding is minimal, i.e. $N_C(2, 10) = 12$. The second equality in (2) corresponds to an orbit of the binary dihedral subgroup $D_2$. The subgroup $D_4$ yields $N_C(2, 8) = 10$. Finally, the earlier known equality $N_C(2, 6) = 6$ (see [K]) follows from an action of the binary tetrahedral subgroup.


Ergodic transforms associated to general averages
Francisco J. Martín-Reyes

Jones and Rosenblatt started the study of an ergodic transform which is analogous to the martingale transform. We present a unified treatment of the ergodic transforms associated to positive groups induced by non-singular flows and to general means which include usual averages, Cesàro-$\alpha$ averages and Abel means. We prove the boundedness in $L^p$, $1 < p < \infty$, of the maximal ergodic transforms assuming that the semigroup is Cesàro bounded in $L^p$. For $p = 1$ we obtain that the maximal ergodic transforms are of weak type $(1, 1)$. Convergence results are also proved. some general examples of Cesàro bounded semigroups.

On distributionally irregular vectors
Vladimir Müller

Let $T$ be a bounded linear operator acting on a Banach space $X$. A vector $x \in X$ is called irregular if $\sup \|T^n x\| = \infty$ and $\inf \|T^n x\| = 0$. The notion was introduced by Beauzamy and is closely connected with hypercyclicity of vectors.

We consider a related notion of distributionally irregular vectors. A vector $x \in X$ is called distributionally irregular if there exist subsets $A, B$ of natural numbers with upper density 1 such that $\lim_{n \in A} \|T^n x\| = \infty$ and $\lim_{n \in B} \|T^n x\| = 0$. Both irregular and distributionally irregular vectors were studied in the context of dynamical systems (under the names of Li-York chaos and distributional chaos, respectively).

(joint work with A. Peris, A Bonilla and T. Bermudez)

Ergodic theorems: from amenable groups to amenable actions
Amos Nevo

We will explain a new general approach to ergodic theory for non- amenable groups, developed in recent joint work with Lewis Bowen. This approach constitutes a natural generalization of the arguments used in classical amenable ergodic theory, namely it employs asymptotic invariance, the doubling condition, Wiener’s covering argument and Calderon’s transference, in the context of non-amenable group actions.
Large Deviations for Sequences of Operators and Maximal Functions
Joseph Rosenblatt

Consider a sequence \((T_n)\) of bounded linear operators on \(L_1(X, \beta, m)\). For \(f \in L_1(X, \beta, m)\), let \(f^*\) denote the maximal function \(\sup_{n \geq 1} |T_n f|\). For various sequences \((T_n)\), we describe some best possible results for convergence of series of the form

\[
\sum_{n=1}^{\infty} m\{|T_n f| \geq w_n\} \quad \text{and} \quad \sum_{n=1}^{\infty} m\{f^* \geq w_n\}
\]

where \((w_n)\) is an increasing sequence of positive real numbers. Types of sequences \((T_n)\) that we consider include ergodic averages, Lebesgue derivatives, martingales, and related classical operator sequences.

Entropy and growth-rate of periodic points
for algebraic actions of \(Z^d\) and more general groups
Klaus Schmidt

For expansive algebraic actions of \(Z^d\) (and, in fact, of residually finite discrete amenable groups) it is known that the logarithmic growth rate of periodic points is equal to the entropy. In the nonexpansive case this question is largely unresolved and touches upon a variety of open problems (including diophantine approximation of curves and approximation of \(L^2\)-Betti numbers).

Quenched limit theorems
Dalibor Volný

Let \((X_i)\) be a Markov Chain and \(g\) a measurable function on its state space. A limit theorem valid for the process \((g(X_i))_i\) with respect to the stationary measure is said "annealed". If it remains true for almost every starting point, it is said "quenched". Every process can be represented as a functional of a Markov Chain and the notion of quenched limit theorems can be expressed by use of conditional probabilities only. We will study several "classical" limit theorems from this point of view.
From ergodic averages with rates to separated nets
Barak Weiss

A separated net is a discrete subspace $Y$ of Euclidean space $\mathbb{R}^d$, such that any point of space is bounded distance from $Y$, and there is a lower bound on distances of distinct points in $Y$. There are natural constructions of separated nets using dynamics of $\mathbb{R}^d$-actions. In 1998, answering questions of Furstenberg and Gromov, Burago-Kleiner and McMullen showed that there are separated nets which are not bilipschitz to $\mathbb{Z}^d$. Burago-Kleiner asked whether such separated nets can arise via minimal toral $\mathbb{R}^d$-actions, and made a connection with rates of convergence of certain Birkhoff averages. In recent work we use this to show that almost all separated nets arising from toral actions are bilipschitz to $\mathbb{Z}^d$. We also analyze the finer equivalence relation of bounded displacement after dilation.

I will summarize some work of Yaar Solomon and report on Joint work in progress with Alan Haynes and Michael Kelly.

The isomorphism problem for ergodic group actions
Benjy Weiss

A fundamental problem in ergodic theory is to classify actions up to isomorphism. I will survey some recent results that formalize the intuition that this problem is a difficult one. It remains difficult even if we restrict attention to ergodic smooth diffeomorphisms of the torus that preserve Lebesgue measure. On the other hand, if we restrict attention to the rank one transformations, which form a generic set, the problem simplifies.

Some problems on recurrence, almost everywhere and mean convergence of single and multiple averages
Máté Wierdl

Lately, many conjectures for subsequence ergodic theorems have been solved, but also, many have fallen. It seems useful to stop panicking, and realize, many interesting new problems have come up as a result of these vigorous attacks. In my talk, I focus on these unsolved problems, but also offer an overview of the recent developments.
Operator ergodic theory for holomorphic mappings

Jaroslav Zemánek

In this joint work with Simeon Reich and David Shoikhet, (local) retractions onto the fixed point set of a holomorphic mapping are studied by means of ergodic properties of the Fréchet derivative at a fixed point.